

AKSF NEWSLETTER

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Association Kangourou

Sans Frontières



Joanna Matthiesen
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***Hello and welcome to
our second Kangourou
sans Frontières
Newsletter.***

I am delighted to write for our 9th AKSF newsletter. Thank you to all AKSF members who helped with newsletter ideas and material. We have a great number of nice articles and interesting topics to read.

It was so nice to see everyone in North Macedonia – didn't we all have a great time? Thanks, Aleksa, for hosting the last AKSF conference, especially under such unusual circumstances. I was happy to learn Aleksa is feeling well and back to his strong health. At the meeting, I especially appreciated the personal connections and talking face to face, starting and fostering relationships, and developing ideas and topics together. My favorite moment at the conference was getting to know new association members, reading the many amazing and high-quality questions, and enjoying the magnificent Lake Ochrid. The special evening that showcased the slideshow of photos from the last 30 years of the Association was priceless and calls for reflection on how far we have come.



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This year is special. We celebrate another successful Kangaroo year – a great accomplishment of those who work very hard. It is an event that started more than 30 years ago and has become a worldwide success in igniting the love of mathematics. This year marks the 30-year anniversary of the death of a great mathematician and Kangaroo promoter, Peter Joseph O'Halloran. Read about his life and accomplishments here:

<http://www.wfnmc.org/obitpoh.html>.

Wishing all of you a wonderful Kangaroo Day. We plan to write again in June. Remember it doesn't have to be long, 2 pages or up to 700 words is plenty. The deadline to submit articles for our May edition is May 1. Let me know if you have a topic to bring to my attention as I graciously await your article ideas and proposals.

*Joanna
AKSF Newsletter Editor in Chief*

News from The President

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Dear Kangaroo friends,

I trust you've all had a good start to the New Year. As winter settles in here in Zurich, my mind drifts back to the sunny days in Ohrid. Our collective effort was rewarding, and I'm delighted about the thought-provoking questions we've selected for the competition. As I write, these questions are being translated into numerous languages. Come March, they'll be embraced by millions of children worldwide.

Have you spotted that we listed the proposing countries in the solutions PDF? You might find this information useful for incorporating into or around the competition within your country.

As you know, one of my visions for AKSF is to share the beauty and the joy of mathematics with all the children in the world, in particular it would be wonderful to have more members in Africa. This was one of the reasons we set up the ASKF4D scheme and a few new members were able to attend our meeting in Ohrid because of the financial support we were able to offer. I think this is tremendously important. To understand our Association and our goals it is important to talk to us and to be with us and the perfect place for this is our Annual Meeting. So again, I would like to ask all those members that may have some savings, to consider donating to AKSF4D (you can ask Robert Geretschläger if you don't know how to donate). If we stand together, we may be able to support even more members and introduce them to our culture and to our family so that they can become our friends and spread our ideas and beliefs.

So, what happened since our Fall meeting in

Ohrid? First of all, the problems we selected were finalised. Thanks a lot, to all who made this happen. It's a lot of work, it happens behind the scenes, and we should all be very grateful to those people who do this job. So

THANK YOU!

And here a few things that were set up since our Fall meeting and things that we have been working on:

- Last year we created a public website <http://www.aksf.org/publications.xhtml> where members can share links to articles that they have written about Kangaroo. Please let us know if you have any papers that should be added here. We hope to establish this site as a site with lots of interesting findings about our competition. Please contact Luis Caceres luis.caceres1@upr.edu

- This year we have started to create a list of books containing collections of Kangaroo questions:

<http://www.aksf.org/publications.xhtml>.

If you want your books listed here, we are happy to add them, please contact Vanek Vladimir vladimir.vanek@upol.cz

- We are also working to establish a research forum. As soon as we have more news, we will inform you. And please let us know if you are interested to help setting this up. Please contact me, akveld@math.ethz.ch.

As mentioned above we have set up a fund to collect money to support members with financial difficulties. It is called

AKSF4D and the money donated here will be used to support members to attend our meetings. More info about how to apply for financial support will follow in a later Email.

- As is probably well known by now, we have set up an WhatsAppGroup for Country representatives. We will use this to inform you about important mails that have been sent or deadlines that are coming up. If you haven't joined us yet, you can use the following link to join the group

<https://chat.whatsapp.com/HkVVPFjJTsnGEHcUafiMPP>

- I think we all very dearly remember the visit of Valentina Dagiènè, the president of the Beaver competition. We share so many common goals that it would be wonderful to see many questions in our (and their) competition about beavers and kangaroos - ideally collaborating with and not competing against each other :-)

Please send me any pictures of Beavers and Kangaroos if you produced them for your competition.

- And we are trying to make us more visible in social media. If you have any news you want to share with us, please mail it to

aksfnews@gmail.com.

Finally, I want to express my gratitude to all the writers who contributed articles, and especially to the editors Joanna, Özgür, and their team for their invaluable efforts in ensuring everything runs smoothly. Your contributions and your work are greatly appreciated!

Feel free to reach out to Joanna if you have any ideas for articles. Whether you're uncertain or eager to discuss a potential topic, Joanna is available. In our vast association, it's challenging to connect with everyone, but our Newsletter serves as a platform for everyone to explore each other's work. It may spark inspiration, initiate discussions, foster collaborations, or even more possibilities.

And don't forget to regularly visit and follow us on Facebook <https://www.facebook.com/aksf.org> and Instagram

https://www.instagram.com/aksf_org

And then there is all the usual stuff, finances, supervising our applicants and provisional members, etc..

Take care, stay healthy!

Yours,

Meike

AKSF President

Follow us on Facebook and Instagram

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AKSF WORKSHOP COMING UP THIS SPRING!

PREPARING GOOD PROPOSALS

This informative workshop is for anyone involved in the math kangaroo problem creation work. It is not just for the country representatives. Please share this information and encourage other team members to participate.

**Saturday, May 4, 2024
@ 14:00-17:00 CEST**

Zoom Meeting Link:

<https://ethz.zoom.us/j/65330978871>

Meeting ID: 65330978871

Speakers: Dennis Ho Christiansen (Denmark)

María Luisa Pérez-Seguí (Mexico)

Alexander Unger (Germany)

We hope to see you and your team there!

AIMO Prize

G r e g o r D o l i n a r

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On November 17, 2023 XTX Markets launched a new \$10mn challenge fund, the Artificial Intelligence Mathematical Olympiad Prize (AIMO Prize). The fund intends to spur the development of AI models that can reason mathematically, leading to the creation of a publicly-shared AI model capable of winning a gold medal in the International Mathematical Olympiad (IMO). The grand prize of \$5mn will be awarded to the first publicly-shared AI model to enter an AIMO approved competition and perform at a standard equivalent to a gold medal in the IMO <https://aimoprize.com/>

The pace at which more and more AI milestones are being passed is astonishing. It was not until 1997 that the IBM computer Deep Blue bit the world chess champion Kasparov. It took almost 20 more years for the computer to beat humans in 2016 in an even more complex strategic game GO, which is considered a major challenge for AI. And although it seems like ages, it has only been a year since the release of ChatGPT completely shook the world and left individuals and humanity reeling, as it has become clear that artificial intelligence has become intelligent - not in some special situations, but in everyday life. AI is already capable of writing excellent speeches, analyzing projects, writing term papers, it is also very good at writing computer code, AI is even not bad at art, for example, producing stunning paintings or music. Soon it will be difficult to find areas in which humans are still significantly better than AI.

One of these areas that we will probably have to wait a little longer for is solving mathematical problems. Not just some

computational problems, but mathematical problems with abstract concepts involving long chains of precise logical reasoning, requiring the highest cognitive functions of human beings. The IMO problems are an excellent example of this. To solve them, you do not need any university knowledge of mathematics, no integrals, no differential equations, no matrices. What you need are very clever ideas, abstract thinking, rigorous reasoning, sometimes also out of the box ideas and in the end a clear presentation of the solution. The AIMO challenge seems to be quite difficult, so it is a very good idea that the AIMO prize also awards intermediate steps on the way to the final AIMO prize, i.e., AI gets a gold medal at the IMO, which means that AI has to solve between 4 and 5 out of 6 IMO problems. At the IMO, more than 600 best pre-university students from around the world are solving six problems, two of which are not too difficult and should be accessible to more than half of the students, two problems are of medium difficulty, accessible to less than a quarter of the students, and 2 extremely hard problems, so that all six problems are usually solved by only one or two of the more than 600 students. Giving half of the 10 million prize to reward progress could motivate many for whom the challenge would otherwise be too scary.

For me, one of the most interesting things about observing the development around the AIMO prize is whether AI will be able to find and use out of the box ideas when solving problems. It is very difficult for humans to come up with unconventional ideas.

Therefore, they are rare and most of them are not even useful. If AI will be able to somehow find an out of the box idea, then that would mean that AI might be able to find and check many out of the box ideas, and it is not clear how that would affect the solution to the problem. The rapid development of AI and the enormous challenges associated with the dangers of AI are one of the most important issues facing humanity at the moment. Huge investments are behind the rapid development of AI, and therefore companies hide information about how AI thinks and makes its decisions in the mysterious “black box” like a snake hides its legs. All the power of AI and the secrecy of how it works makes people suspicious and fearful of AI. But AI is here to stay. AI is extremely useful, even irreplaceable, and to gain some confidence that it can be kept under control, it is very important to have an open access publicly shared AI, that can solve very difficult IMO problems, one could even say the most difficult problems that a human with pre-university education can solve. For all these reasons, XTX's decision to encourage and reward the development of publicly shared AI is commendable.

It is not surprising that such an important player among trading companies, as XTX undoubtedly is, is interested in mathematics and math competitions for young people. Trading companies do not hire economists or other people with social science knowledge as brokers. They prefer to recruit the best young minds with education in math or computer science, etc. For trading, they need exceptionally sharp, very young minds who are able to make extremely quick decisions and make good estimates with very little data and logical thinking. As with chess, where the best chess players are in their twenties or early thirties, so it is with trading, where it is very rare to be a trader in your late thirties.

I am sure that many people will follow the

progress of the AIMO Prize with great interest, especially people involved in mathematical competitions and wondering when the problems of their own competition will be solved by AI.

Gregor Dolinar
President of the International
Mathematical Olympiad Board

Invitation to WFNMC

Robert Geretschlaeger
robert@rgeretschlaeger.com



An open invitation to all active “Kangaroos” to participate in the upcoming WFNMC Mini-Conference in Sydney, Australia.

If you have been reading the AKSF newsletters carefully, you may recall my article about the World Federation of Mathematics Competitions (WFNMC) in issue 1 of this newsletter in June 2021. (If not, I would suggest taking a look at pages 8-9 of that newsletter.) The WFNMC is an international organization that has been active since 1984. It aims to provide a common platform for everyone interested in Mathematics Competitions of all kinds on a national, regional or international level. Since this certainly describes everyone involved with AKSF, it would be wonderful if even more Kangaroo organizers could find their way to joining in the activities of the WFNMC.

There are many ways to join in these activities. You can simply read the publications of the WFNMC, especially its journal Mathematics Competitions, which is published twice a year, and is available online at <http://wfnmc.org/journal.html>. (Of course, you are also invited to submit articles to the journal for publication.) Perhaps most important of all, you can take part in the meetings of the organization.

The next WFNMC Mini-Conference will take place in Sydney, Australia on July 6, 2023. This will be a satellite conference of the International Congress on Mathematical Education (ICME), which will be held in Sydney from July 7th through July 14th. More information on the WFNMC and the mini-conference is available at <http://wfnmc.org/conferences.html>, and more on ICME is available at <https://icme15.org/>.

It would be wonderful to be able to see you all at the meetings in Sydney! I am certain that you will not regret coming to the WFNMC mini-conference, and I am certain that anyone reading this will find something right up their alley at ICME, where all manner of issues on the teaching of mathematics are presented and discussed, including mathematics competitions. If you have any specific questions about the WFNMC, please don't hesitate to get in touch with me directly at robert@rgeretschlaeger.com.

I hope to see you in Sydney!



From the president's desk STACK

Meike Akveld
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STACK – my favourite question type in Moodle

Disclaimer: This is not an article about Kangaroo, it is not even an article about math competitions. Nevertheless, it may be interesting to those of you that teach or examine large classes or want to give automatic but still individual feedback in learning processes.

You all know me as the president of AKSF, but that's not actually my job. I am a math lecturer at the ETH Zurich, involved in teacher education and in teaching large classes of engineers' basic math (which is actually from all my activities the one I love most) and when time permits, I do research in tertiary education.



The ETH Zurich is a popular university for both Swiss and foreign students. In our bachelor studies, we see many students from Germany and Austria. At the master's level, the student population becomes even more international as the teaching language is predominantly English and the ETH has an outstanding reputation for teaching and research. Figure 1 shows the growth of the student population over the past 20 years and the prediction for the next 15 years. Simultaneously, budget cuts and other uncertainties predict no staff increases whatsoever.

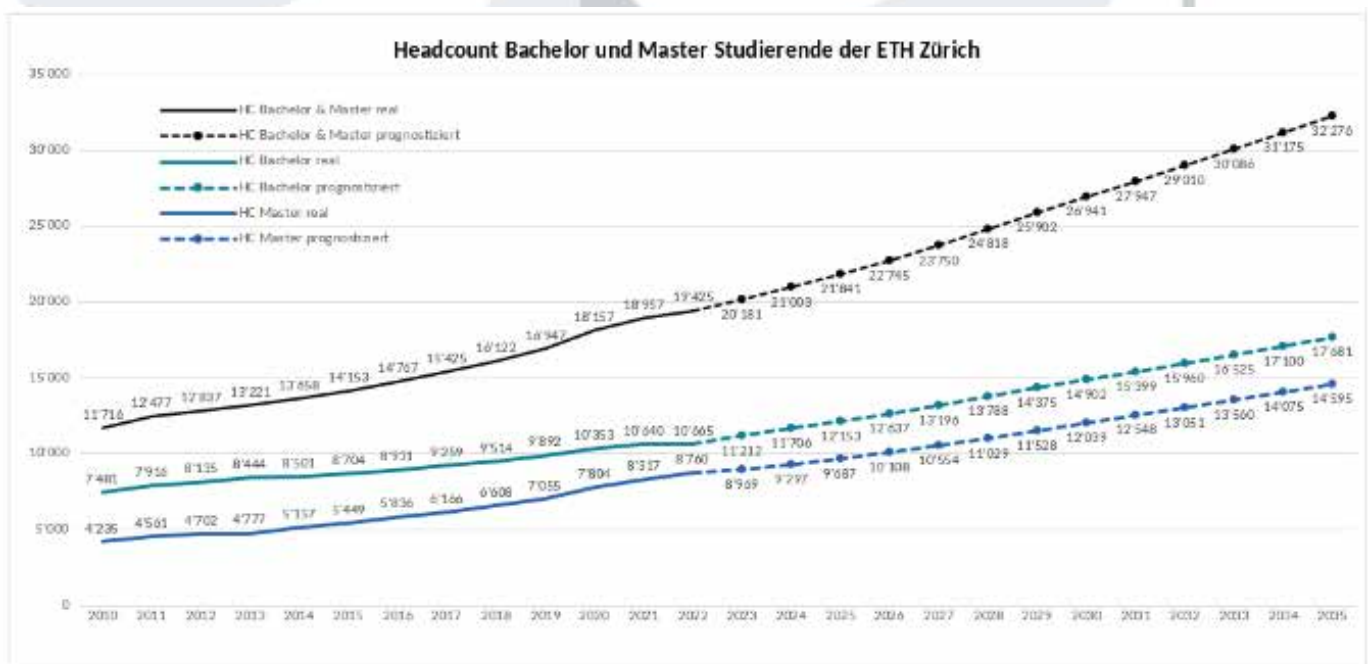


Figure 1: Student Growth at ETH

This causes problems: We struggle with the grading of our exams, and we struggle to give individual feedback to our students – a very important factor for learning, as many studies have shown.

A few years ago, I got to know Professor Chris Sangwin from the University of Edinburg (UK) and he introduced me to the question type STACK, which can be implemented as plugin to Moodle or many other LMSs. For me STACK has two amazing properties:

- it allows randomization at a sophisticated level.
- it allows partial credit or individual feedback because of so-called Potential Response Trees (PRTs).

In this short article I only want to give you a brief glance of what STACK is about. If you are interested, feel free to consult the links at the bottom of this text, or contact me directly.

What is STACK?

STACK is the world-leading open-source automatic assessment system for mathematics and STEM. It is available for Moodle, ILIAS and as an integration into other systems through LTI.

The main features of STACK are the following:

- Students type in mathematical expressions and are not restricted to multiple choice (MCQ).
- Answers are graded based on mathematical properties.
- Answers are validated before they are marked, so students are not penalized for poor programming skills.
- STACK can generate random questions, so students are shown different variants of questions, and can repeat quizzes with new variants.
- Students are given feedback that refers to their specific answer and mistake, as if marked byhand. This is done using the so-called Potential Response Trees (PRTs). The potential response tree is the algorithm which establishes the mathematical properties of the student's answer and assigns outcomes. A potential response tree (technically an acyclic directed graph) consists of an arbitrary number of linked nodes. In each node two expressions are compared using a specified answer test, and the result is either true or false. A corresponding branch of the tree can do each of the following:
 - Adjust the score, (e.g., assign a value, add or subtract a value).
 - Add written feedback specifically for the student.
 - Nominate the next node or end the process.

Some examples

Figure 2 shows an example about the product rule for differentiation – the question is in German, but I am sure you all understand. The student has to determine the derivative of the product at a given point:

Gegeben seien zwei differenzierbare Funktionen f und g . An der Stelle $x_0 = 5$ seien die Werte der Funktionen sowie ihrer Ableitungen gegeben:

$$f(5) = 0, f'(5) = 3, g(5) = 0, g'(5) = 7.$$

Dann ist

$$(f \cdot g)'(5) = \text{[input box]}$$

Prüfen

Figure 2: A typical STACK question

The code of this question is fairly easy and can be seen in Figure 3; the values of the functions and the derivatives vary from 0 to 9 and are randomly chosen, so that a student can repeat the question when it's wrong.

```

/*
Written by Meike Akveld, George Ionita, Andreas Steiger, ETH Zürich
Contact and more information:
stack@math.ethz.ch - www.math.ethz.ch/stack
*/
xx0:rand(10);
yy0:rand(10);
zz0:rand(10)+1;
yy1:rand(10);
zz1:rand(10)+1;
ta:yy0*zz1+yy1*zz0;
false1:zz0*zz1
false2:zz0*yy1-yy0*zz1;

```

Figure 3: STACK code of the above question

Figure 4 shows a typical student answer, a well-known misconception, namely the derivative of the product is the product of the derivatives (wishful thinking). As I, as a lecturer, am aware of this problem, I have programmed specific feedback for any student that makes this mistake. So not only does the program tell the student their answer is wrong and shows a full worked solution, but it also addresses this particular mistake and explains why it does not work.

Gegeben seien zwei differenzierbare Funktionen f und g . An der Stelle $x_0 = 5$ seien die Werte der Funktionen sowie ihrer Ableitungen gegeben: Frage-Taste und eingetragte Variablen

$$f(5) = 0, f'(5) = 3, g(5) = 0, g'(5) = 7.$$

Dann ist:

$(f \cdot g)'(5) =$

✘ Falsche Antwort.

Sie haben wahrscheinlich gemeint, dass $(f \cdot g)'(x_0) = f'(x_0) \cdot g'(x_0)$. Das wäre zwar schön, aber ist leider falsch. Das sieht man schnell mit einem Gegenbeispiel. Wenn $f(x) = g(x) = x$ so gilt $(f \cdot g)(x) = x^2$ und somit $(f \cdot g)'(x) = 2x \neq 1 \cdot 1 = f'(x) \cdot g'(x)$.

Lösung: Sie müssen hier die Produktregel der Differentialrechnung anwenden d.h.

$$(f \cdot g)'(x_0) = f(x_0) \cdot g'(x_0) + f'(x_0) \cdot g(x_0)$$

In diesem Fall gibt das

$$\begin{aligned}
 (f \cdot g)'(5) &= f(5) \cdot g'(5) + f'(5) \cdot g(5) \\
 &= 0 \cdot 7 + 3 \cdot 0 \\
 &= 0.
 \end{aligned}$$

Figure 4: A typical wrong answer and its specific and general feedback

Behind this question is a so-called PRT, see Figure 5. The student's answer moves down this binary decision tree and undergoes a test at every node. In this case e.g., it is compared with the product of the derivatives and if they agree, the relevant feedback will be given.

At the ETH we use STACK questions both in formative and in summative assessment. We have created a large database of questions, from which we can pull the questions that we need for our weekly quizzes and in this way the students can practice as much as they want and get relevant feedback when they make mistakes. A couple of lecturers work together on these databases and in this way we join forces to create a large collection of high quality questions with good feedback.

We also run bonus questions (that count towards the final grade) and actual exams in this system. The randomization helps us to avoid cheating. The PRTs allow us to give partial credit for common mistakes that we know in advance or which we realize after an exam and can then still program.

For me STACK has opened opportunities that I have been thinking and dreaming about for many years. It is not so easy at the beginning, the programming is slightly non-trivial, but not impossible, and especially if the work is done as a team, it's actually a lot of fun.

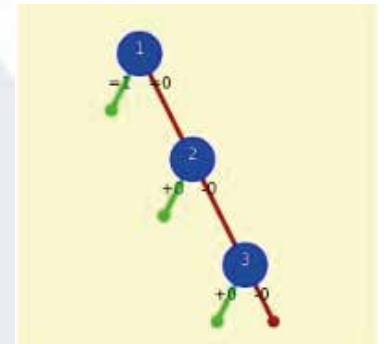


Figure 5: A typical PRT

Links:

STACK Homepage : <https://stack-assessment.org/>

ETH STACK Homepage : <https://math.ethz.ch/mathematik-und-ausbildung/e-learning/stack.html>

Interview about the use of STACK at the ETH :
<https://ethz.ch/en/news-and-events/eth-news/news/2023/09/eth-zurich-employ-computers-as-supplementary-maths-tutors.html>

Meike Akveld

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The impact of Kangaroo numbers in Brazil

Leonardo Cavalcante
Brazilian Math Kangaroo Team
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The Math Kangaroo Competition celebrated its 15th year in Brazil in 2023. Throughout this journey, I have seen an increasing number of students, teachers, families, and schools approaching the competition somehow and transforming us into a big community in Brazil.

2019 was my first Kangaroo year and since then I have noticed how hard everyone at ASKF has worked to improve mathematics learning and present it in a way that resonates with students' daily activities - especially through the problems we propose every year.

Following up on this work has been amazing both in the International Community and in Brazil. Also, seeing how many lives we can impact as well as the growth of Kangaroo in my country throughout the years still makes me feel astonished.

I remember in 2021 when I learned that almost 400k Brazilian students were solving our tests. At that time, it represented a big number for the country, especially after the first year of the pandemic, and made our whole team proud. Finally, this number made Brazil rank first globally among AKSF member countries - I could not be prouder of being part of Kangaroo.

The contest kept growing - in 2022 the number of students was over 700k and reached nearly 1 million last year. It has been growing organically at a rate of about 30% per year, which comes to show we have achieved

important results in the Brazilian education environment. For me it is not more than just numbers, but a sign that we can increasingly improve the educational reality of our community and let teachers and students know they are not alone.

Our latest national indices shows that 95% of our students from public schools (which are the majority in Brazil) finish high school without the expected math knowledge.

Furthermore, only 5% of students in the last year of high school can solve probability and trigonometry problems. The following article details this data further:

<https://exame.com/brasil/95-dos-alunos-saem-do-ensino-medio-sem-conhecimento-adequado-em-matematica/>



On the other hand, Brazil has celebrated great overall placements in important competitions, such as the International Mathematical Olympiad (IMO) - 16th place in 2023, with 6 medals including a gold one - a Fields medal, and the development of cutting-edge research in mathematics.

How can we analyse such contradictions? And why are they happening? Among many factors, one must consider some issues faced by primary and secondary schools: Brazil lacks public policies to improve teaching quality, especially mathematics; the Brazilian math curriculum is too extensive in terms of content and not suitable for the school calendar; teachers suffer from the training issue since graduation; mathematics has been worked in an uncreative way.

In this scenario, I believe knowledge competitions are among the most effective initiatives to improve basic education teaching and learning, as well as one of the most suitable in Brazil considering not only the diversity of schools in our country, but also the nature of the Kangaroo tests.

Kangaroo, for example, reaches every Brazilian state, both public and private schools, in big cities and the countryside - it is the largest international math competition in the country. Additionally, the problems in the tests are suitable for all of our students.

Through many discussions with our team and our community in Brazil, I have been more and more surprised with the great capacity of Kangaroo to democratise mathematics, and this is noticeable both in the students - who often ask their schools to register for the competition - and in the teachers - who enthusiastically promote the competition and do everything they can to apply it within their schools.

The Kangaroo questions can be described as formative and the large majority of them have multiple entry points, so they are accessible to all students. At the same time, they can also be solved at higher levels.

Because of this richness we received a lot of requests from teachers and schools for materials to help them work on these kinds of activities with their students.

With that in mind, our team started providing Kangaroo lesson plans on our website.

Also, last year we launched “Canguru Experience”, a platform for teachers with Kangaroo content for the classroom - lesson plans, workshop scripts, continuing education, and more. From 2024 onwards, we are structuring the platform into tracks suitable to the main needs of Brazilian schools.

Through surveys within our community, we learned that 94% of schools believe Kangaroo is very effective in engaging students,

87% of them have noticed an effective engagement and learning growth in students’ learning, and 67% of the teachers who used our problems in the classroom felt more stimulated and appreciated.

Kangaroo has proven to be a yearlong initiative in Brazil and, for students, a vibrant celebration of mathematics. Our goal is to help as many teachers as possible to reinvigorate the school environment with the problems of the contest throughout the school year and to approach math contents creatively and safely.

Finally, I would like to share with you a testimony that warmed our hearts. The report comes from Marlon, a public-school student from Rio de Janeiro suburb: "My name is Marlon Fagundes Pereira Junior. I am 19 years old, and I have participated in Kangaroo Competition since 2017, winning 4 gold medals and 1 silver medal. I would like to share that I have been approved to study in the USA on a 100% scholarship (...) I am going to study a double major in mathematics and computer science. Kangaroo represents part of my journey, and these results were part of my application. I grew up in a favela in Rio de Janeiro, and your work helped open this precious opportunity for me.

So, I would like to give you my report and my sincere thank you!".

This fills us with a lot of satisfaction because we have seen it is possible to change the lives of children and teenagers through education and that Kangaroo is an important partner in this journey. We are not alone. We are a community with more than 100 countries that has grown and reached greater numbers worldwide, and I know we will never stop empowering people through math.

In the name of the Brazilian team, I also would like to say Brazil is looking forward to welcoming our entire community to Santos this year. The XXXII Annual Meeting means for us the opportunity to celebrate on Brazilian soil the impact of the work of AKSF on so many students and teachers' lives in our country.

See you all soon!



Leonardo Cavalcante

Brazilian Math Kangaroo Team

Scripta Manent

The purpose of this column is to discuss, periodically, proverbial phrases from philosophy, literature or history that are relevant to Mathematics. In each case, we explore the origin, meaning, and use of maxims which mathematicians and intellectuals often like to refer to.



M i c h a e l L a m b r o u

l a m b r o u @ u o c . g r

Archimedean problem

Archimedes (287-212 BC) was undoubtedly the greatest mathematician of Greek antiquity and certainly one of the most influential of all times. His contributions to Mathematics were pioneering, deep and varied. For example, he devised ingenious methods that are precursors of the Integral Calculus of Leibniz and Newton, which he applied to determine the areas and volumes of various shapes such as the sphere, the paraboloid of revolution and the spiral. He also dealt with Mechanics, Hydrostatics and Astronomy where he made notable contributions. Although some of his works have not survived, fortunately, the most important ones do survive either in the original Greek (namely Measurement of a Circle, The Sand Reckoner, On the Equilibrium of Planes, Quadrature of the Parabola, On the Sphere and Cylinder, On Spirals, On Conoids and Spheroids, On Floating Bodies, Ostomachion, The Method of Mechanical Theorems) or in later Arabic translations (Book of Lemmas etc).

We shall not elaborate on his contributions

here. Instead, we shall only be concerned with the history and interpretation of the proverbial phrase "Archimedean problem" that has been used since antiquity in Greek and Latin literary texts, not necessarily mathematical, as a metaphor for a particularly complex problem.

The earliest mention of the phrase is in Latin and is due to the great Roman scholar, philosopher, author, statesman and lawyer Marcus Tullius Cicero (106-43 BC), whose influence on Latin language and literature was immense. Cicero received part of his education in Greece itself, which was at that time a province of the Roman Empire, and was well acquainted with the Greek language and Philosophy.

In two letters to his lifelong friend Titus Pomponius Atticus (110 – 32 BC), Cicero used the expression "Archimedean problem" to describe difficult or intricate situations. The letters were written in Latin but the phrase "Archimedean problem" is quoted in Greek. The two occasions, in translation, are: "The encomium to Cato which is expected of you is nevertheless a

truly Archimedean problem" and "The Archimedean problem which I assigned you to solve, and which previously worried me, has now been resolved."

But where did the phrase "Archimedean problem" gain its reputation from?

It is known, and we shall see examples, that Archimedes used to send letters from his hometown Syracuse to the mathematicians of Alexandria, where he had studied in his youth (he was a student of the students of Euclid). Some of these letters contained the enunciations of his new, unpublished, theorems but they were not accompanied by their proofs. The reason was that he wanted to give the mathematicians of Alexandria, which was the centre of Greek Mathematics at the time, the opportunity to investigate them on their own before supplying, at a later date, his latest manuscript which contained the proofs. For example, in the preface to his book "**On Spirals**" he writes to his friend the mathematician Dositheos, referring to an earlier letter he had sent to Conon, who had in the meantime deceased, the following, which I translate from the original Greek text:

"The statements of theorems I sent to Conon, for which you always urged me to write the proofs [...] I am now sending these proofs to you, after writing them in the present book. But don't be surprised that it took me such a long time to publish their proofs because the following happened: I first wanted to show them to those involved in Mathematics who wished to investigate them on their own". Similarly, in the initial paragraph of his then-new book "**On the Sphere and Cylinder II**", sent to Disitheos, Archimedes wrote the phrase "previously you asked me

to send you the proofs of the problems whose statements I have earlier sent to Conon..."

The problems that Archimedes posed in his letters were difficult and in any case a challenge even to the first-rate Mathematicians of Alexandria. An examination of those that have survived till now, and there are several, convinces us of this. After all, he often mentions in his books that no one seemed to have solved the problems he had sent some time earlier. In his own words in "**On Spirals**", just before writing the proofs of his new results, he states "[...] since many years have passed and I have not been informed that anyone has solved the problems, I wish to explain each one to you [...]"

For the solution to Archimedes' problems, it was not enough for his contemporary mathematicians to simply find clever arguments to deal with them. To tackle them one had to find paths that used deep ideas unknown at that time, but only known to the great master himself. For example, finding the surface area of a sphere with which Archimedes challenged the mathematicians of Alexandria, entailed discovering techniques that belong to the then unknown Integral Calculus. Namely, taking the liberty of using an anachronism, let me mention that in modern language and notation, his method was effectively an ingenious geometric calculation for finding the integral of $\sin(x)$ using Riemann sums! This was well ahead of his time.

Occasionally the problems of Archimedes also contained deliberate traps. We shall look at two examples.

a) In one of his letters Archimedes sets the problem of proving that the area of the spherical zone between two parallel planes that intersect a sphere is proportional to the square of the width of the zone. Correct? Can the reader prove this on his own, even using integrals?

Of course not, because the correct theorem is counterintuitive stating that the area in question is proportional to the width of the zone and not to the **square** of the width, as erroneously stated! Archimedes knew this but, as he writes, for some theorems he purposely did not formulate the correct statement in his first letter because some people in Alexandria had the bad habit of stealing his theorems, saying that they themselves had discovered them in the past. This way he caught them red-handed.

He characteristically writes in one of his books, when he later sent the proofs of the statements he had announced in his previous letter, that "[...] because it so happened that I have added here incorrect statements of two of my theorems, so that those who claim to have found them all, but without presenting any proof, are exposed as having discovered the impossible".

b) There is yet another type of trickery that Archimedes incorporated into his problems. An example of it appears in a Greek manuscript discovered in 1773 by the philosopher and dramatist Gotthold Ephraim Lessing in the remarkable Bibliotheca Augusta at Wolfenbüttel, in Germany. It contains the so-called **problema bovinum** or **Cattle problem**

of Archimedes in the form of a beautiful poem of 44 lines. This was sent in a letter by Archimedes to his friend Eratosthenes and the Alexandrian Mathematicians, challenging them for its solution.

The problem asks to find the number of oxen and cows of various colours in a herd, given some simple numerical interrelations. It leads to a system of 8 equations with 8 unknowns, 7 of which are linear and the 8th is a Pell equation (which is a quadratic). Nothing seems easier, yet in the last verses of the poem Archimedes says "if you succeed in expressing all the numbers of the sought magnitudes, go boasting that you have emerged victorious knowing that you have been judged as being perfect in this kind of wisdom (i.e. skill with numbers)".

This last verse seems provocative. Although an experienced mathematician can easily compile the required system of equations and in principle would not have any difficulty in solving them, it seems as if Archimedes was saying that the solver would not succeed. What is the catch? Where is the trap that Archimedes set for the unsuspecting would-be solver?

The answer is that Archimedes has masterfully chosen the data of the problem so that the answer is such a large number that it is impossible to write it down in full. Indeed, it was only in 1965 and with the use of powerful computers that it became possible to record the full answer on paper. It turns out that the number of bulls and cows consists of 206544 digits, which means that our entire solar system is too small to accommodate them.

Such is the story of the phrase

“Archimedean problem”, which was used in literary texts as a metaphor for a complex situation that needed attention. It reflects the habit of the great mathematician to pose next-to-impossible problems with which he challenged his contemporaries.



Painting by Benjamin West (1738 – 1820) showing Cicero discovering the tomb of Archimedes. The incident is described by Cicero in his *Tusculanae Disputationes*.

M i c h a e l L a m b r o u

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Findings from the survey of the members of the Association Kangourou sans Frontières

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A survey was conducted of the Kangaroo membership in 2023. The survey of the AKSF membership saw responses from almost 60 countries, with all regions being well represented.

We know that there are large differences between regions, and the findings should not be seen as prescriptions: these responses reflect local circumstances including things like the quality of the postal services, internet access, poverty levels and things like access to printing resources (paper and printers) which varies greatly.

It was encouraging to see that in answer to the question regarding motivation where we offered the following ideas

helping to develop a love of mathematics amongst learners

- showing learners that mathematics can be fun
- showing learners that mathematics can be useful
- helping to stimulate problem solving skills

the most common comment was along the lines of “All of the above!”

We present 10 noteworthy findings:

The respondents believe AKSF will have 10 million participants in 2030. This means doubling over the next 7 years. Much of that growth is currently anticipated to come from Brazil. The Brazilians see their participation rates rising to 5 million by 2030.

Their rapid growth in recent years will be due to a combination of factors:

- they use many marketing channels
- they charge a school a fixed price to participate (rather than an individual) so that once a school signs up, they are not disincentivized narrowing the field of participants)
- they offer differentiated offerings (eg with or without training material) and
- they charge for medals separately to keep the base price low.

Many have plans for the future ranging from adding a digital edition, offering more languages, offering more preparation and practice functionality, and doing more marketing.

In terms of difficulty, most don't amend the papers leaving them unchanged (75%) although 20% make the papers easier and 5% make the papers harder. This may be because many don't know that the papers can be amended, or that the difficulty level can be changed. With that said, the papers may be at the right level for the current audience.

Many countries were interested in running a second round with 1 in 5 already doing so, and a third wanting to do so in future. This may be a way to make the event more accessible, say with easier questions being used in the first round, and harder questions being saved for a second round. This would be a significant change, and hard to roll out globally as an additional round would mean additional effort and expense, but...

Many organisers (60%) already run more than one event over the year (not always maths related),

related), and many (two thirds) are interested in running additional events

About 40% of countries only run their event on paper (with most posting the papers although many email the papers to the teachers), just under 10% only run the event online and more than half of the respondents run the event both online and on paper.

This impacts many things from the cost (and entry fee required to cover costs) to the number of languages that can be made available and whether or not marking is automatic, and whether qualification for a second round could be easily facilitated by automated marking and reuse of the same platform.

When the event was first held and the resource characteristics of the country seem to determine the approach. For example, when looking at the respondents by when they first held the event:

3 of the first 10 to participate (1994-6) participate both online and offline, with 7 out of 10 offering either an email or paper posting options and

only 1 out of the last 10 to join (2022-3) offer only a paper based participation option

A case can be made that online participation is more viable for older learners with teenagers being more likely to have devices (e.g. tablets and smartphones).

Half of the respondents distribute detailed solutions, although a third share an answer key with a non-trivial number not sharing solutions. The solutions are shared in different ways, with some making the solutions available online after the event, with some publishing books with detailed solutions and related inspiring content.

Almost everyone offered access to past papers as training material.

Cheating is seen as a risk by many. Invigilation by teachers is widespread, and some use centralized venues or run the event at the same time (not just on the same day). When solutions are shared is also a factor, with many doing so after the test window.

Many countries offer learners a way to participate in multiple languages, including French, German, Russian and Spanish.

1 in 4 respondents indicated that they are interested in letting learners participate in pairs.

There is a large variation in what the event organisers charge and offer learners. Some offer prizes, certificates, medals, toys, training material, books, printed papers etc... Certificates could be either digital or physical, and in some cases they were personalised and were only sent to those achieving a certain score.

The prices seem to reflect the cost of the underlying offering which varies by country, with many who are running online events (or receiving state funding or sponsorship) able to have free participation: more than a quarter charge nothing, a third charge €1 or less and the median entry price was €2.50. Of those that collected fees, the most common payment collection methods were bank transfers/ETFs and credit/debit cards. It was most common for schools to collect the entry fees per learner, although just under 10% charge a fee per school (independent of the number of learners)



Participation fees vary greatly, but so do the offerings

Event organisers promote their event in many different ways, and at times this will reflect local conditions (e.g. privacy policies and whether learners can be contacted directly). This table ranks the avenues by our estimate of the importance of the different avenues, with a website, social media and emailing teachers and learners being regarded as the most important.

	Very important	Moderately important	Important	Unimportant	Don't do	Importance
Website	71%	18%	9%	0%	2%	1.00
Social media (eg Facebook, Instagram)	64%	18%	5%	2%	11%	0.90
Email to teachers	59%	23%	9%	5%	4%	0.86
Email to learners	30%	11%	18%	11%	30%	0.45
Viral marketing	18%	16%	16%	5%	45%	0.33
WhatsApp	16%	16%	7%	11%	50%	0.27
SMS	13%	14%	13%	13%	48%	0.23
TV	9%	9%	20%	13%	50%	0.17
Newspaper	5%	14%	25%	11%	45%	0.17
Radio Messages	7%	7%	11%	14%	61%	0.12

Many use many platforms to market their event

It should be noted that the results are skewed towards the active members, and away from the inactive members whose difficulties may not be adequately reflected by this survey.

We will look to publish more detailed findings and analysis in a journal in 2024, and to run a follow-up survey (hopefully in 2025) identifying trends. We will again identify and share best practice and thinking so that global participation can be increased.

Greg Becker

Interactive League, An Experience in Iran

Sepideh Chamana

Iran Math Kangaroo



The interactive league is a problem-solving challenge for elementary school students. This league is held online in Iran, for four days, in the days before or after the Kangaroo match in April. In this league, students solve about 10 problems -2 or 3 problems every day-. These problems are selected from Kangaroo 2016 to 2020.

Why and how interactive?

Elementary school students enjoy hands-on activities more than pen and paper. In case to give students the chance to enjoy solving math problems, we thought to rebuild some of the problems of the previous years of the Kangaroo match, which could be converted into one of the interactive facilities of the "Bebras" Platform. So students can solve some problems by dragging and dropping the answers, or by clicking on pictures, and so on, and enjoy doing so.

To generate this league, the following steps were made:

1- First of all, I should have known various interactive types of problems in Bebras and I had to get acquainted with all kinds of facilities in that platform. These are:

- Maze: the problem is solved by a series of arrows or commands.
- Puzzle: the problem can be solved by drag and drop.
- Imap: by clicking, the case (for example, the color) will change. It has more than two cases.
- Btn: by clicking, the case (for example, the color) will change. It has only two cases.

Short response: the answer to the problem is written in a box.

2- Then, I reviewed all the problems of the Pre-ecolier, Ecolier, and Benjamin groups from 2016 to 2020. By doing so, I found common contexts between problems. Besides I realized the problems that could be restored in one of the various types of interactive forms.

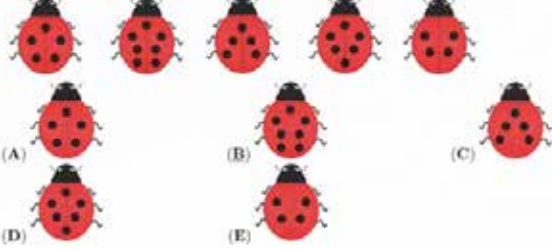
3- According to the common contexts that I had seen on the problems of each age group, I chose these contexts from first to sixth grade in elementary school:

- The adventures of the garden of friendship and their loved ladybugs and butterflies and other insects.
- Bees and butterflies and other insects.
- A hypothetical house and the courtyard and different rooms of it.
- A hypothetical land of toys.
- A magic Garden and animals in it,
- A Kitchen and edible.

4- For each grade, I chose about 10 to 12 problems. The problems were usually chosen from kangaroo 2016 to 2020, but there are few problems from other years. I chose the problems in such an arrangement in which the students could see a story in them. So a slight change in the characters or description of some problems (to match the selected context for that grade) was done: redesigning some problems without any change in their mathematical nature.

5- After that, the technical work phase began. The graphics of problems were changed by a graphic designer. Then each problem was converted into an interactive problem. So, the set of problems in each grade was made. You can see some examples below:

6. Which of these ladybirds has to fly away so that the rest of them have 20 dots in total?



Picture 1. The original version of problem: PreEcolier, 6-2018



Picture 1-a. Redesigned Puzzle version of problem for grade 1. Students answer this problem by dragging the selected ladybird and dropping it on the flower. You can see the trace of the computer mouse in the picture.



Picture 1-b. The answer of the problem.

12. A number is written on each petal of two flowers. One petal is hidden.



The sums of the numbers on the two flowers are equal. What number is written on the hidden petal?

- (A) 0 (B) 3 (C) 5 (D) 7 (E) 1

Picture 2. The original version of problem: PreEcolier, 12-2020



Picture 2-a. Redesigned Puzzle version of problem for grade 1. Students answer this problem by dragging the selected number and dropping it on the petal which is hidden by the ladybird.



Picture 2-b. The answer of the problem.

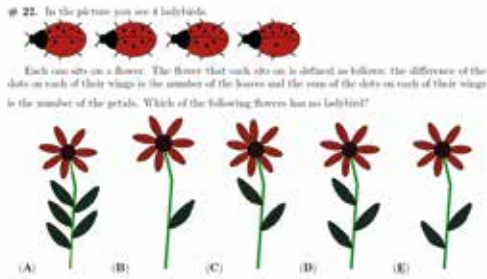
Hint:

The following numbers are used in the problems and they are written in persian numeric.

- ۱ = 1 ۲ = 2 ۳ = 3 ۴ = 4 ۵ = 5 ۶ = 6
۷ = 7 ۸ = 8 ۹ = 9 ۱۰ = 10 ۱۱ = 25



Picture 3. The redesigned Puzzle version of preEcolier, 20-2005, in Persian.



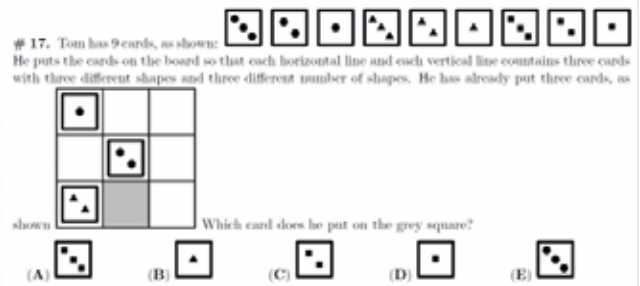
Picture 4. The original version of problem: PreEcolier, 22-2016



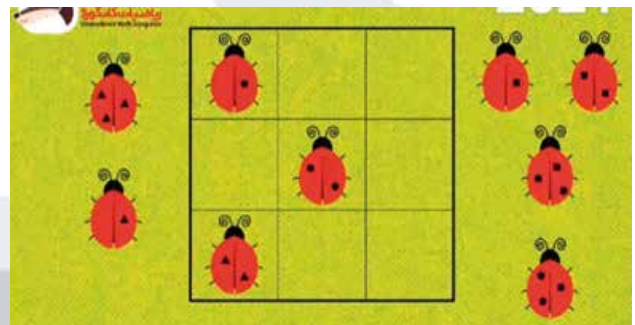
Picture 4-a. Redesigned Puzzle version of problem for grade 1. Students answer this problem by dragging the selected ladybird and dropping it on the corresponding flower.



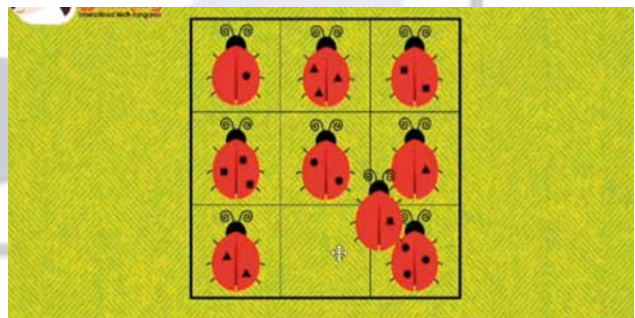
Picture 4-b. The answer of the problem.



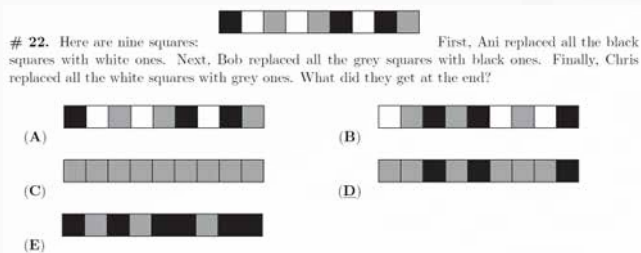
Picture 5. The original version of problem: PreEcolier, 17-2020



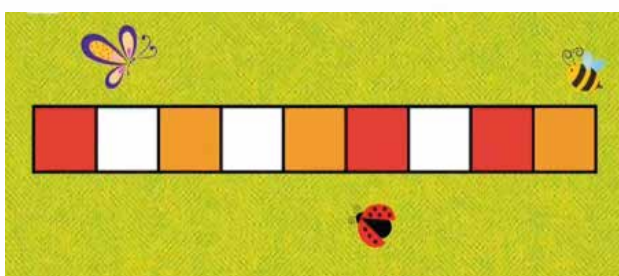
Picture 5-a. Redesigned Puzzle version of problem for grade 1. Students answer this problem by dragging the selected ladybird and dropping it on the correct square of table.



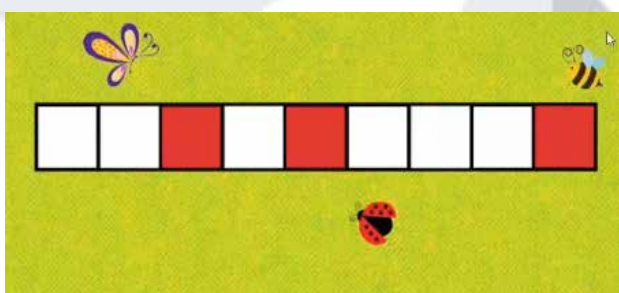
Picture 5-b. The answer of problem. You can see the trace of the computer mouse in the picture.



Picture 5. The original version of problem: PreEcolier, 22-2019

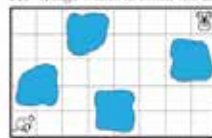


Picture 6-a. Redesigned Imap version of the problem for grade 1. Students answer this problem by clicking on each square and changing its color by the instructions of the problem.



Picture 6-b. Student changes all red squares to white. And now, he/she is changing orange squares to red.

11. Kangas wants to reach the koala without going through any of the coloured squares.



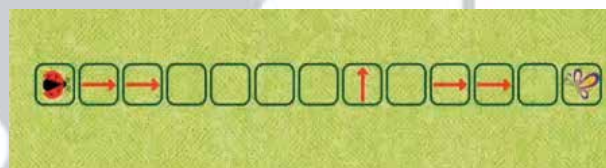
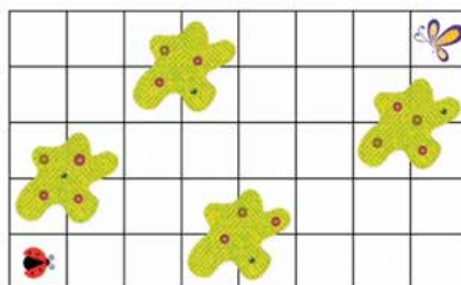
Which route could she take?

- (A) (B) (C) (D)

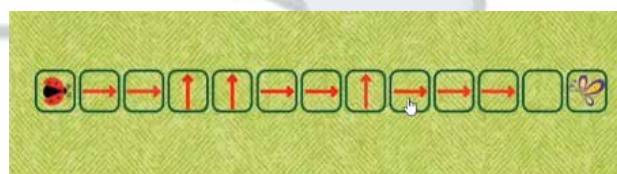
Picture 5. The original version of problem: PreEcolier, 11-2022

باغ گل‌ها کششورگ می‌خواهد به یوزپانه برسد، بدون اینکه از مربع‌هایی که در آن‌ها چمن کاشته شده بگذرد. کدام مسیر را می‌تواند برود؟

برگردان ها در جاهای خالی، مسیر را کامل کنید.



Pictures 7-a & 7-b. Redesigned Btn version of problem for grade 1. Some direction arrows are shown. Students must solve this problem by clicking on each empty square and selecting the correct arrow.

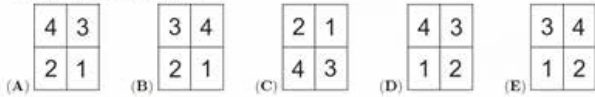


Picture 7-c. Student is selecting the correct direction arrow for each square.

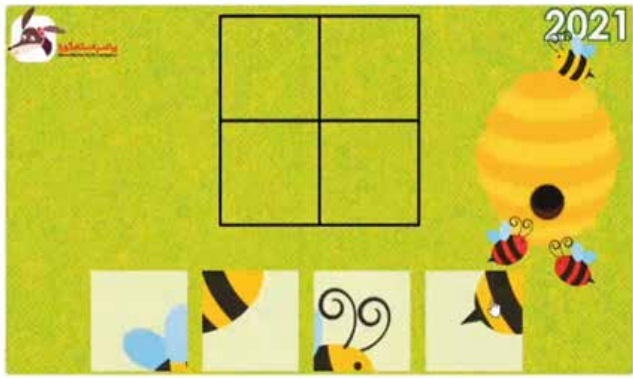
3. Nelly arranged the 4 pieces to make a picture of a kangaroo.



How are the pieces arranged?



Picture 8. The original version of problem: PreEcolier, 3-2020



Picture 8-a. Redesigned Puzzle version of problem for grade 2. Students answer this problem by dragging the pieces of picture and dropping each of them on the correct place.



Picture 8-b. Student is solving the problem. You can see the trace of the computer mouse in the picture..

To change some problems into interactive one, there was some combinatorics needed. For example, the problem in picture (3), has $\binom{3}{2} \binom{3}{1} = 9$ correct cases that must be given to system to check the correctness of students' answers.

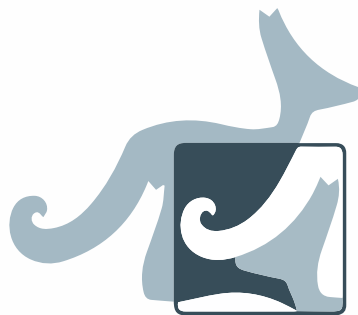
This redesign experience was very enjoyable for me and I lived with kangaroo problems during these days when I was involved in doing this project.

Sepideh Chamanara

Happy Math Kangaroo Day

for all

Thursday,
March 21, 2024



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