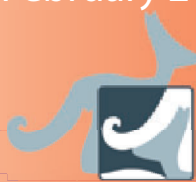


Issue 12
February 2025



Association Kangourou
Sans Frontières



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Hello and welcome to our 11th Kangourou sans Frontières Newsletter.

Dear Kangaroo Friends,

Welcome to the latest edition of the Association Kangourou sans Frontières Newsletter. As always, we are delighted to bring you updates and insights from our vibrant mathematical community.

In this issue, we feature a range of articles contributed by our dedicated members, showcasing innovative approaches to mathematics education and sharing experiences from recent events. Your contributions are the heartbeat of this newsletter, and we extend our heartfelt thanks to each author for their invaluable input. THANK YOU, THANK YOU, THANK YOU!

What's new and exciting? One event that stands out is the collaboration between the US and Canadian Math Kangaroo teams. Together, we facilitated a workshop for teachers—our first-ever open to participants from over 20 countries. Ildiko, the presenter, brought her energy and warmth to the workshop, teaching us how to make the most complex topics accessible and enjoyable for every student. Her expertise in problem-solving strategies engaged teachers in small group discussions after the main session. This open workshop tradition started two years ago, and we are thrilled to see it appreciated and enjoyed. We exchanged ideas that deepened our



Joanna Matthiesen
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understanding of how to use Kangaroo questions to teach effectively. A big thank you

to Ildiko and everyone who attended this past fall. Looking ahead, we are excited to host a seminar this February with the esteemed Professor Daniel Finkel. If you wish to participate, feel free to [enroll](#).

As we approach the Math Kangaroo 2025 Competition on March 20, 2025, I want to wish all of you a wonderful and enjoyable experience. May this event inspire a deeper appreciation for mathematics, foster curiosity and learning, and mark another milestone as we approach 7 million attendees.

Warm regards,

*Joanna
AKSF Newsletter Editor in Chief*

News from The President

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Dear Kangaroo friends,

I hope you all had a good start into the New Year. Did you know the following fun facts about 2025:

- $2025 = 729 + 1296$ and this is the only way it can be written as the sum of two squares;
- It is a polite number, since it can be written in 14 ways as a sum of consecutive naturals, for example, $403 + \dots + 407$;
- 2025 is an equi-digital number, since it uses as much as digits as its factorization;
- Adding to 2025 its reverse (5202), we get a palindrome (7227).

and I am sure many of you know many other nice properties of 2025. It's a nice number and I hope it will be a good year!



Switzerland is currently blanketed in snow, and I find myself reminiscing about the sunny days we spent in Santos, Brazil. I also think about the great work we accomplished together and the wonderful set of questions we created! These questions are now being translated into many languages across the globe, some of which we may not even be familiar with. By March, they will be solved by millions of children once again.

As you know, one of my main goals for AKSF is to share the beauty and joy of mathematics with children everywhere, and it would be especially fulfilling to have more members from Africa. This was a key reason behind the creation of the ASKF4D scheme, which allowed some members from Africa to attend our meeting in Santos thanks to the financial support we, together with an anonymous generous donor, could provide. I truly believe this is crucial. To



fully understand our Association and its goals, it's essential to engage with us directly, and the best opportunity for this is at our Annual Meeting. Therefore, I'd like to ask those members who might have the means to consider contributing to AKSF4D (if you're unsure how to donate, please reach out to Robert Geretschläger). By standing together, we can support even more members, introduce them to our culture and family, and help them become our friends, spreading our ideas and values.

What has happened since our Fall meeting in Santos? First and foremost, the problems we selected have been finalized. A huge thank you to everyone who contributed to making this possible. It's a significant amount of work that takes place behind the scenes, and we should all be deeply grateful to those individuals who handle this important task.

THANK YOU!

And here a few things that were set up since our Fall meeting and things that we have been working on for longer:

- Have you spotted that we listed the proposing countries in the solutions PDF? You might find this information useful for incorporating into or around the competition within your country to show how international we are.
- As mentioned above we have set up a fund to collect money to support members with financial difficulties. It is called AKSF4D and the money donated here will be used to support members to attend our meetings. More infos about how to apply for financial support will follow in a later Email.
- I'm sure we all fondly remember the visit of Valentina Dagienè, the president of the Beaver competition. We share so many common goals, and it would be fantastic to see many questions in both our (and their) competitions featuring beavers and kangaroos – ideally working together rather than competing against one another! :-). If you have created any images of beavers or kangaroos for your competition, please do send them my way.
- And we are trying to make us more visible in Social Media. If you have any news you want to share with us, please mail it to aksfnews@gmail.com. And don't forget to regularly visit and follow us on Facebook <https://www.facebook.com/aksf.org> and Instagram https://www.instagram.com/aksf_org
- And then there is all the usual stuff, finances, supervising our applicants and provisional members, etc....

Lastly, I would like to extend my heartfelt thanks to all the writers who contributed articles, and a special thank you to the editors, Joanna, Özgür, and their team, for their immense effort in making sure everything runs seamlessly. Your contributions and hard work are truly appreciated!

If you have any ideas for articles, don't hesitate to get in touch with Joanna. Whether you're unsure or excited to explore a potential topic, she's here to help. In our large association, it's not always easy to connect with everyone, but our Newsletter provides a great opportunity for us all to discover each other's work. It could inspire new ideas, spark conversations, encourage collaborations, or lead to many other possibilities.

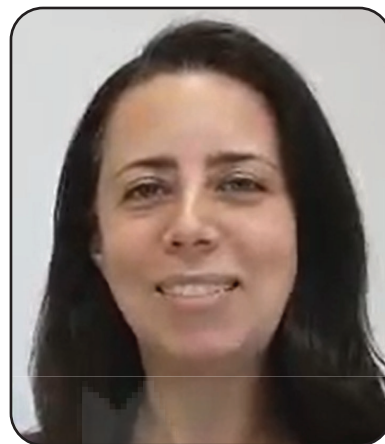
Take care and stay healthy!

Yours,

Meike

AKSF President

Planning and Executing AKSF Conference



Cristina Diaz
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Since 2018, we knew Brazil would host the AKSF Annual Meeting. However, the pandemic delayed our plans, and only now, in 2024, have we finally made it a reality. Organizing an event of this scale for participants from all over the world was a first for us, which brought us to challenges and valuable lessons along the way.

One of our initial challenges was selecting the right location. Brazil offers many incredible cities, and Salvador, in Bahia, was a strong candidate. However, it was challenging to find accommodations that matched our needs. We wanted to provide a comfortable experience for all participants, ensuring you would not need to travel far for the event activities while also offering a suitable place for rest and meals. Santos was ultimately chosen for its practicality—meeting rooms located within the hotel and the beach just a block away provided a convenient and pleasant setting.

The possibility of rain was another concern. Brazil had faced heavy rains in 2024, and we wanted to ensure that this would not affect the event. We rented generators for the work and assembly rooms to guarantee power in case of outages and planned an indoor activity as an alternative for rainy days. Fortunately, the weather cooperated, providing us sunny days for participants to enjoy the city, despite rain during the evenings.

Additionally, it was our first time dealing with bureaucratic matters related to the entry of international participants into Brazil. Usually, we are familiar with the process of obtaining visas for other countries, but not for our own. It was surprising to learn about the documentation required for some participants to enter Brazil. We worked diligently to assist everyone in meeting these requirements and ensuring a smooth entry process.

A standout moment for us was reuniting our team. As we work remotely from different parts of Brazil and the world, opportunities for in-person meetings are not common. Typically, only a few representatives attend international meetings. This time, seeing all our mathematics teachers together at the event was both meaningful and rewarding.

Finally, the closing night was a memorable moment. We wanted to celebrate in true Brazilian style, with lots of energy, relaxation, and great food, creating a joyful experience for everyone on the last day of the gathering. By the way, the mix of Brazilian samba and math T-shirts came as the best-unexpected thing that could happen.

This event was a significant milestone for us, and we are proud to have successfully brought it to life. We hope all participants had a positive experience and left with fond memories.



Kangaroo without borders: a friendship story

Maria Luisa Perez-Segui
Kerstin Jordaan
Tatevik Adamyan
Maria Elizabeth Losada

There are some words that can describe the Kangaroo annual meetings, some are **Mathematics for everyone**, but two others can be friendship and travel. This year in Santos, some of us ended our kangaroo meeting by staying an extra day. We were four women: Maria Luisa from Mexico, Kerstin from South Africa, Tatev from Armenia and Maria Elizabeth from Colombia.

We only found out that we were staying an extra day on the last day of the meeting. First, the two Maria's from Colombia and Mexico found out just by chatting. We work in the same group (Student), we've known each other since the nineties and have a lot of competitions in common. At some point of the excursion day we realized we were staying an extra day and when we did, as we went on separate tours to the coffee museum, we were already looking to see what tour we could take the next day - together!



The tour guide on the double-decker bus, Diogo, was very interesting and I, Maria Elizabeth, decided to ask him for a personal tour, so after some exchange of whatsApps, by the end of the day he had given us an interesting 4-hour option.



That evening, during the Gala, I went down to try to book an extra night at the hotel and I met Kerstin. She was trying to get some vouchers accepted at the check-out desk. That's when I found out that she was also staying a day, so I invited her on the tour and she said that her friend Tatev was also staying the night. So then we were 4.

The next day, Diogo picked us up at 1pm for a 4-hour tour around Santos. We had already discovered the day before that coffee and the port had developed Santos. That Santos had a port on the very protected side of the Island, really solved the mystery for me as to why we could see from the beach big ships coming in, but no visible port for them to go to. Just that stop opened up the history and geography of Santos for us all!



This time we went to San Vicente, which is basically the town next door on the island. Then to the orchidarium which is a fancy name for a public zoological and botanical park. But there were some orchids.



And then to a secret stop which was Pele's memorial monument. I really was a fan of Pele!



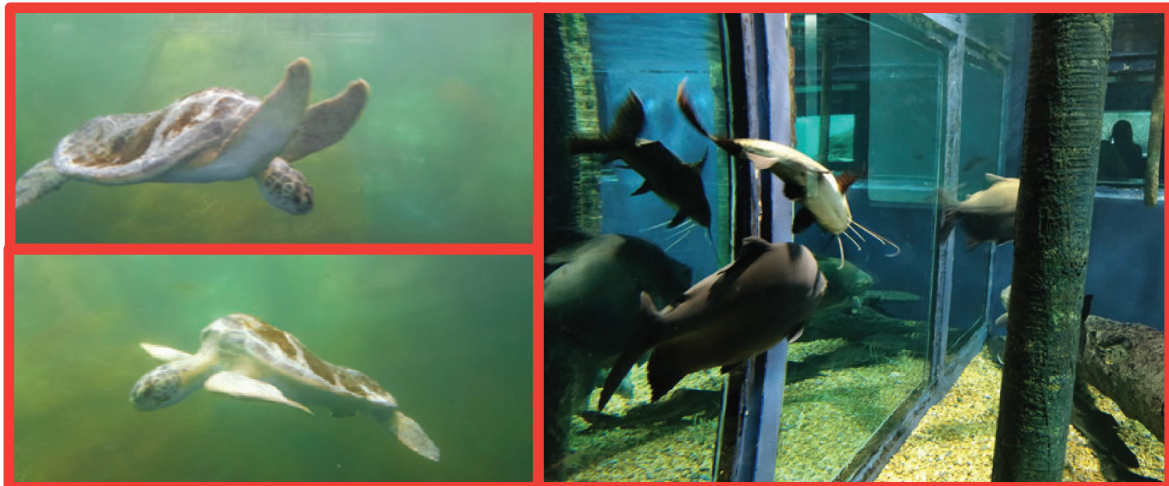
We continued on to the Monte Serrat up the funicular. You may remember, if you were on the same bus, that it was inspired by the Montserrat in Barcelona that has “la morenita”. Oddly enough back home in Bogotá, my partner was at the same time planning to climb our own Monserrate too.



We drove around the city center and the city in general, getting to know Santos and finally understanding where it was that we had been. The port system with its customs building, warehouses and trains was a whole side of the island, here there is a great movement of coffee, orange pulp, cellulose and others of Brazil's thriving export industry.



One of our last stops was the Aquarium.



Diogo had said that at the end of canal 2 at a kiosk on the beach there was the best Caipirinha of all Brazil and from all the awards from around the world the cocktail expert had, he wasn't exaggerating that much (Brazilians do have a way of describing things they are talking about as the “best in the world”, maybe they only talk about things that are that good). We ended our day of touring in the vicinity of the hotel. There at the caipirinha kiosk with “the rabbit” Rogerio, since Rogerio Coello means Roger rabbit, well, creatively. The drinks and mariscos were very nice! Some of us had more than one to make sure...



We had the perfect ending to a lovely day at the mall by the hotel, where Kerstin had discovered some very promising looking ice cream place. And it delivered!



Maria Luisa and Kerstin continued to Sao Paulo the next day. Tatev and I went to GRU together to return home.

Thank you, Elio, Cristina, Pablo, Laura and all the Brazilian team for organizing this Kangaroo meeting in Santos.



Till we meet again!

Calibrating a competition

Mike Clapper and Andrew Kepert



Mike Clapper
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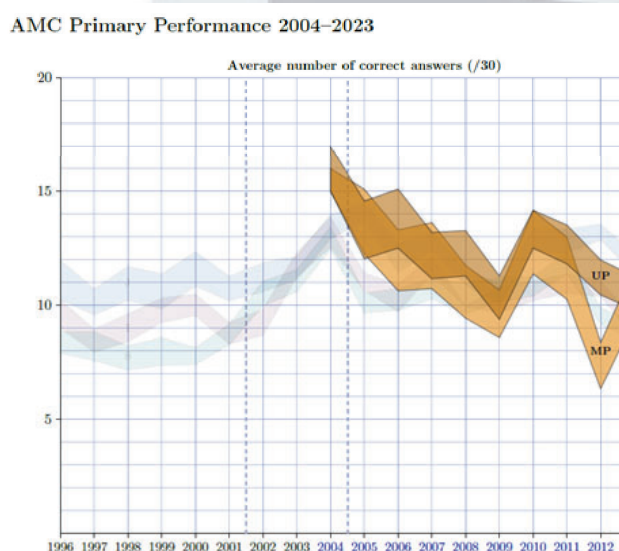
This paper outlines the efforts made over the last 10 years to ensure a consistency of difficulty of papers in the Australian Maths Competition, a competition very similar in nature to the KSF. It is based on a talk given by the authors in the WFNMC mini-conference in Sydney in July, 2024.

Mike Clapper is Chair of the AMC problems committees (both primary and secondary) and currently Chair of the Student problem committee for KSF.

Andrew Kepert is a lecturer in Mathematics at Newcastle University, NSW and is the problems co-ordinator for the AMC committees and principal architect of the database described.

The Problem

In 2012, we knew we had a problem. We started the primary divisions of the Australian Maths Competition (AMC) in 2004 and, despite a very stable committee, by 2012 we found that the average number of questions students were getting right was decreasing to an unacceptably low level, with Year 3 students averaging just six correct questions out of 20, compared with 15 in 2004. Results in the other primary year levels showed a similar decline. The graph below shows the change in performance over these 9 years. MP represents Middle Primary, years 3 and 4, with the upper line being the performance of Year 4, whilst the lower line shows Year 3 performance. Similarly, the ribbon for UP (Upper Primary – Years 5 and 6) shows performance for those two levels.



At around the same time, we were getting feedback from a number of schools that the secondary papers were also getting harder and that, in particular, questions were taking too long to work through, so that even students who were in IMO training were unable to complete the papers in the time allowed (75 minutes). We were also finding that, on occasion, questions were placed inappropriately in the paper (which was supposed to get harder as you worked through it) and certain types of question appeared to be very poor discriminators.

The first national AMC was held in 1978 and the competition grew very quickly so that there were more than 500,000 competitors by the early 90s. The introduction of State and National testing had some negative impact on these numbers, but the competition retained a very strong reputation, both in Australia and internationally. It was (and still is) the cornerstone of the Australian Maths Trust (AMT) which allowed us to fund our Olympiad programs and produce high-quality student resources for developing problem-solving skills. Any negativity associated with the competition had to be taken seriously.

The Cause

It is difficult to know with certainty why competition papers were becoming more challenging but one factor appeared to be ‘committee creep’. That is, with a very stable committee structure, we become so used to the types of questions on offer that we see them all as routine and gradually ratchet up the difficulty level without being aware of it. I have seen this occurring in other competition committees, so whilst stability is an asset in many regards, it can also become a problem. Associated with this, teachers often see ‘process’ questions as straightforward and ‘puzzle-type’ questions as more challenging, but for students it is often the reverse. Students with logical capacity can often work through puzzles much better than we expect and then fall down on what we regard as routine mathematical procedures.

AMC committees are a mixture of teachers and academics or working mathematicians. We have two separate committees (with a small overlap) for Primary and Secondary divisions. Each division covers two years of school and match the divisions used for KSF (but without Pre-colier). Each paper has 30 questions, the first 25 of which are multiple choice (1 – 10, 3 marks; 11-20, 4 marks; 21-25, 5 marks). The last 5 questions have a numerical answer 0 – 999, and are successively worth 6, 7, 8, 9 and 10 marks, giving a maximum possible score of 135 marks.

The Solution

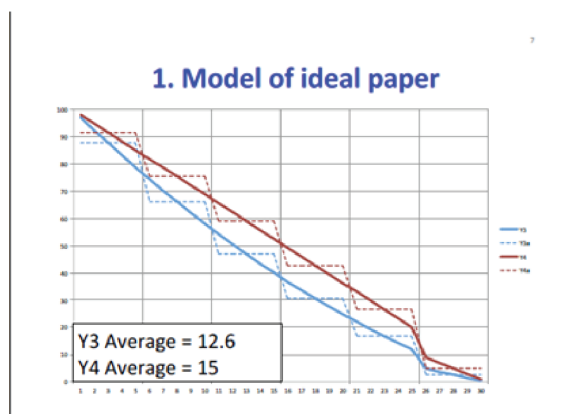
We needed to do something about it or risk reputational damage, reduction in numbers and loss of income. So we put a proposal to the AMT Board. This proposal consisted of several elements:

- Defining the ‘ideal’ or model AMC paper
- Developing a method for predicting question success
- Understanding the characteristics of questions that perform well/badly
- Using the above insights with committees
- Testing the model and methodology against competition results and continually improving

We shall look at each of these in more detail.

The ideal paper

Every competition serves a different purpose and audience, so the ideal paper for one competition may be significantly different from that for another. For instance, in comparison with KSF, the AMC probably starts with easier questions but finishes with significantly harder questions. In developing



our model paper, we decided that, for the upper age group within each division, we wanted the average number of questions correct to be around 15 out of 30. We wanted almost everyone to be able to get the first question right, around 20% to get the final multiple choice question right (question 25) and less than 1% to get the final question right. When we prepare papers, we work in blocks of five questions, so we do not expect great variation within each block of five. This is reflected in the dotted lines on the diagram.

Predicting question success

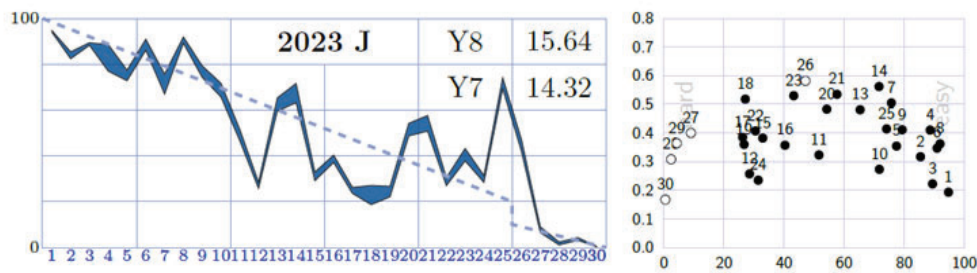
We have always maintained some data on competition question performance. This includes the percentage of correct responses, the distribution across distractors and a discrimination index for each question. However, this was difficult to access within the problem committee setting. So we have created a database which contains data for all previous questions from 2004 to date. For each paper, questions are organised in order of difficulty, graphs are available for every paper to show variations from the model paper and the database is easily searchable to find comparison questions for any question under consideration. The database is very large but is easily navigable by the use of embedded pdf hyperlinks. When looking at new questions, we can usually identify and easily access questions on a similar topic and which are of similar complexity. This enables us to predict the success of new questions and guides us as to where they should be placed within the paper.

Understanding the characteristics of questions that perform well/badly

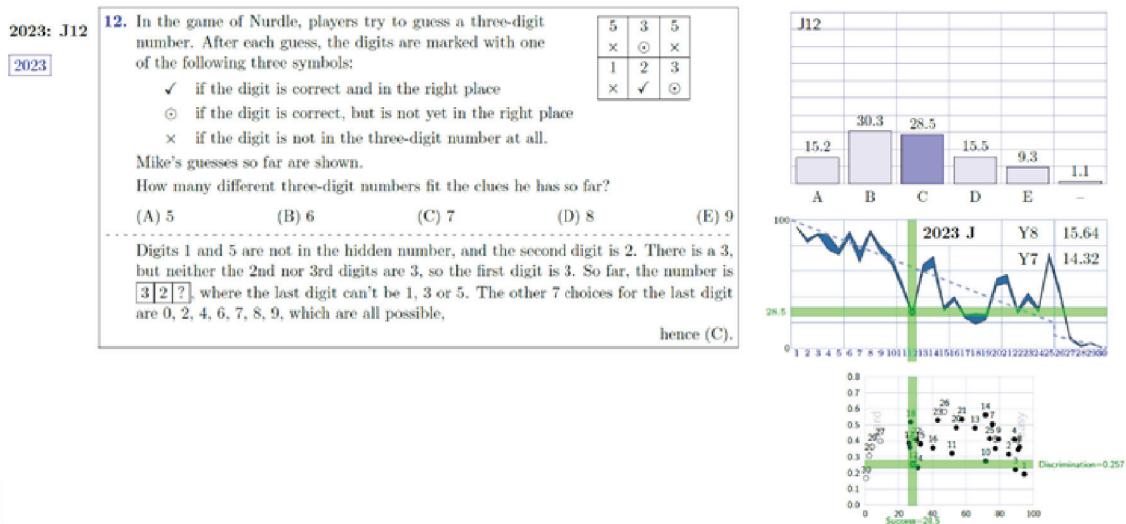
Using the database reveals, over time, questions that are habitually misclassified. It has also taught us about types of questions that discriminate poorly against student overall performance. An example of this is the type of counting question which asks to count the number of triangles or rectangles in a given diagram. Whilst these questions are excellent investigative questions for students to tackle, in our view they do not perform well as competition questions.

Using the above insights with committees

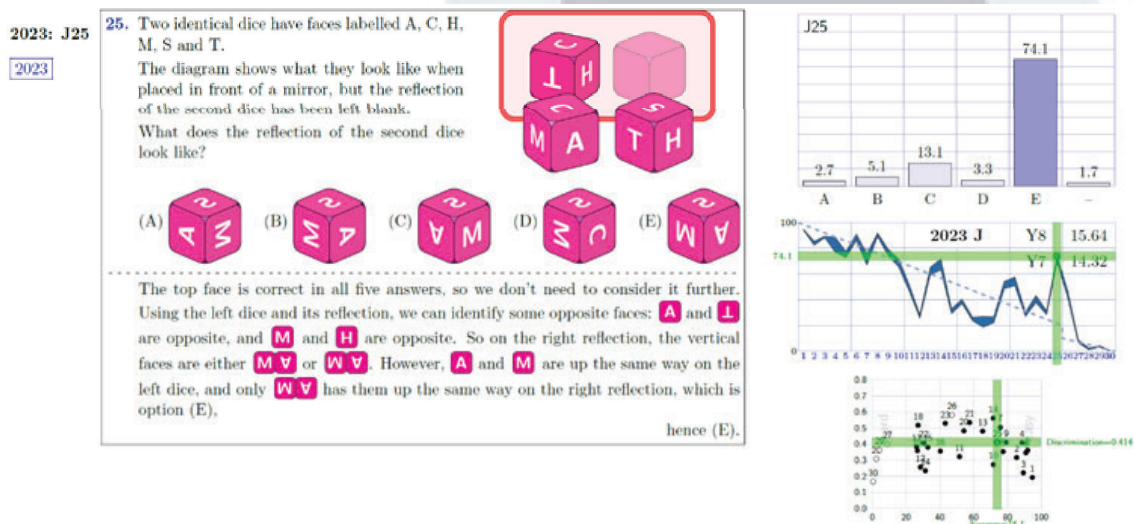
Before commencing on a new set of papers, the committee looks at the performance of the previous year's paper, focusing on anomalous questions. This provides insight into which types of question and which parts of a paper are working well, and where we need refinement or reflection. As an illustrative example, here is the graph of the 2023 Junior paper (Years 7 and 8) which the committee would have accessed when meeting in 2024 (to prepare the 2025 paper).



We can see that overall, the paper performed well, with average correct responses in the desired range. The beginning and end of the paper was very close to the model line. However, there were a number of anomalous questions, for example, question 12, which underperformed and question 25, which overperformed. We can click on question 12 and go straight to the question, revealing the following:



Discussion would be around the effect of distractors (in this case, distractor B was compelling, probably because students forgot that 0 was a possible digit), the discrimination (this question had a fairly low discrimination index) and in this case, the fact that the question had low inherent feedback (ie – it is not obvious to the student when they have reached an answer that it is right or wrong).



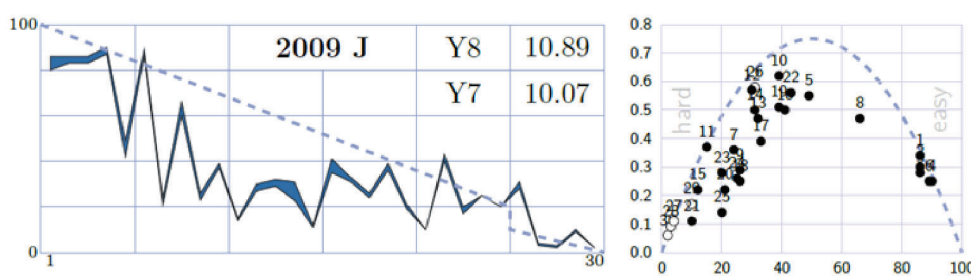
Here we see that the question was about 3-D visualisation and the students did much better than we expected, perhaps because teachers can find these questions quite hard! However, this question does have a much higher discrimination index and also higher inherent feedback, so it was a successful question, even if it should have been placed earlier in the paper.

More broadly, we also observe that the middle of the paper was generally a little harder than intended, so we take this into account when setting future papers.

When we start working on new questions, we prepare in advance a spreadsheet of comparison questions drawn from the database, which the committee can consider when determining where to place each particular question.

Testing the model and methodology against competition results and continually improving

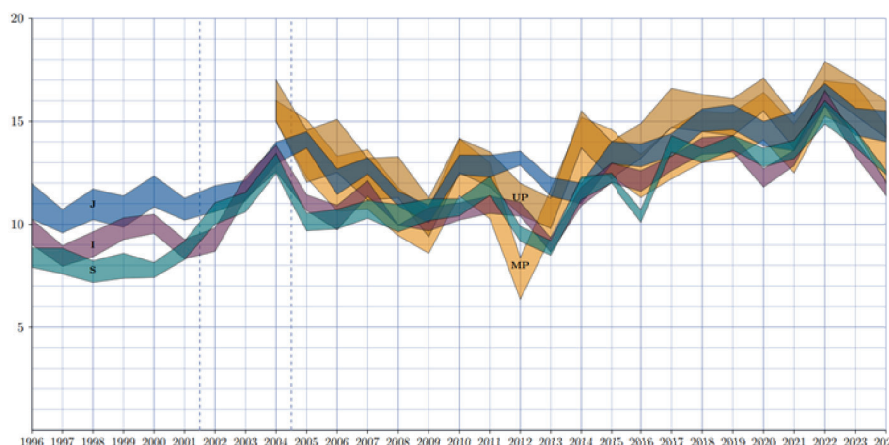
We never expect to get every question right. However, over the years, we hope to continually refine our methodology and conform more closely to our desired model. If we take an earlier example of a paper, such as the 2009 Junior paper, you can see that we have come a long way in getting a paper



which achieves what we set out to do.

This is most clearly shown in the graph below, which gives average performance for all papers from 1996 up to 2024.

AMC Performance 1996-2024



Since we started the project in 2013, we have seen a gradual but significant improvement in consistency within and between papers and we have avoided the disasters which occasionally befell us in earlier years. The problems committees also have much more meaningful discussions around the effect of distractors, where we have a readily available evidence base to support our theories.

This paper was originally designed as a conference presentation, so we cannot show the ease with which we can use the database in the same way as in that setting. However, hopefully it has provided evidence of the usefulness of the database and food for thought on competition design.

Matjaž Željko awarded Erdős Prize

The AKSF IT expert Matjaž Željko was recently honored for his groundbreaking work in creating and developing internet-based technologies for mathematics competitions. The prestigious Erdős award was presented to him at the mini-congress of the WFNMC (World Federation of National Mathematics Competitions) in Sydney, Australia, in July 2024.

His work is fundamental to our selection of problems for the Kangaroo competition and organizing our meetings, as well as to the IMO (International Mathematical Olympiad) and dozens of academic competitions in his native Slovenia.

Congratulations to Matjaž for this well-deserved recognition!



Matjaž Željko (right) receiving his award from Kiril Bankov (Chairman of the WFNMC Awards Committee, left) and Robert Geretschläger (WFNMC President, center).



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Call for Publications and Book Titles related to Math Kangaroo

The AKSF invites contributors to submit details of their scholarly articles and books (including problem collections) for publication on our AKSF website. Showcase your or your team's academic work and share your knowledge with the AKSF community!

For Articles:

Please provide the following information:

- Year of publication
- Authors
- Title
- Language
- Short abstract
- Name of journal and publisher
- Volume and journal number
- Page numbers (if applicable)
- ISSN or ISBN
- URL (if available online)

For Books:

Submit the same details as for articles, except the abstract is not required.

We look forward to highlighting your contributions and fostering a vibrant exchange of ideas within the AKSF network.

Contact Luis with the scholarly article listing request at: luis.caceres1@upr.edu

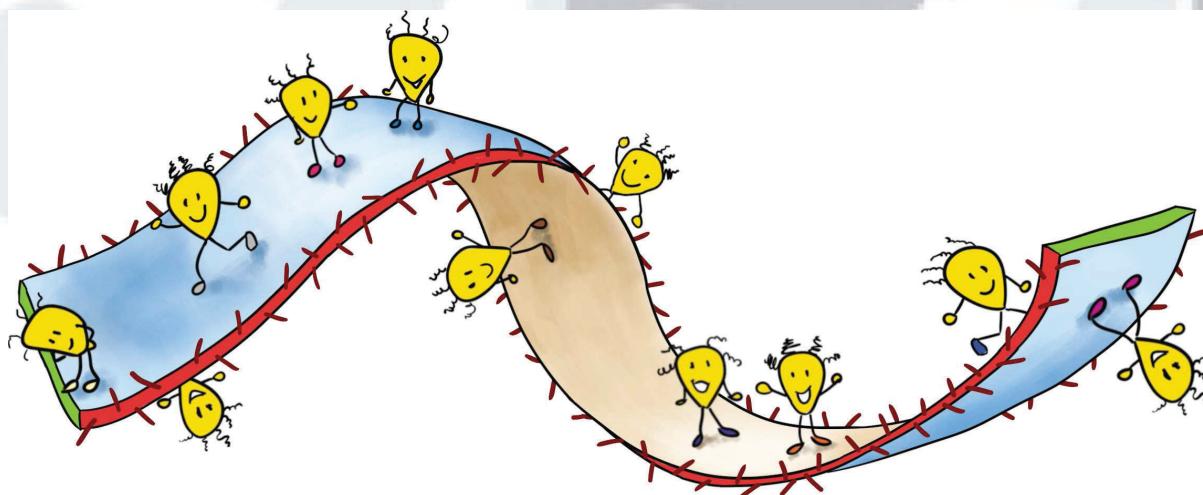
Contact Vladimír with the book listing request at: vladimir.vanek@upol.cz.

A fairytale about a Möbius strip

Łucja Mentzen
ilewicz@wp.pl

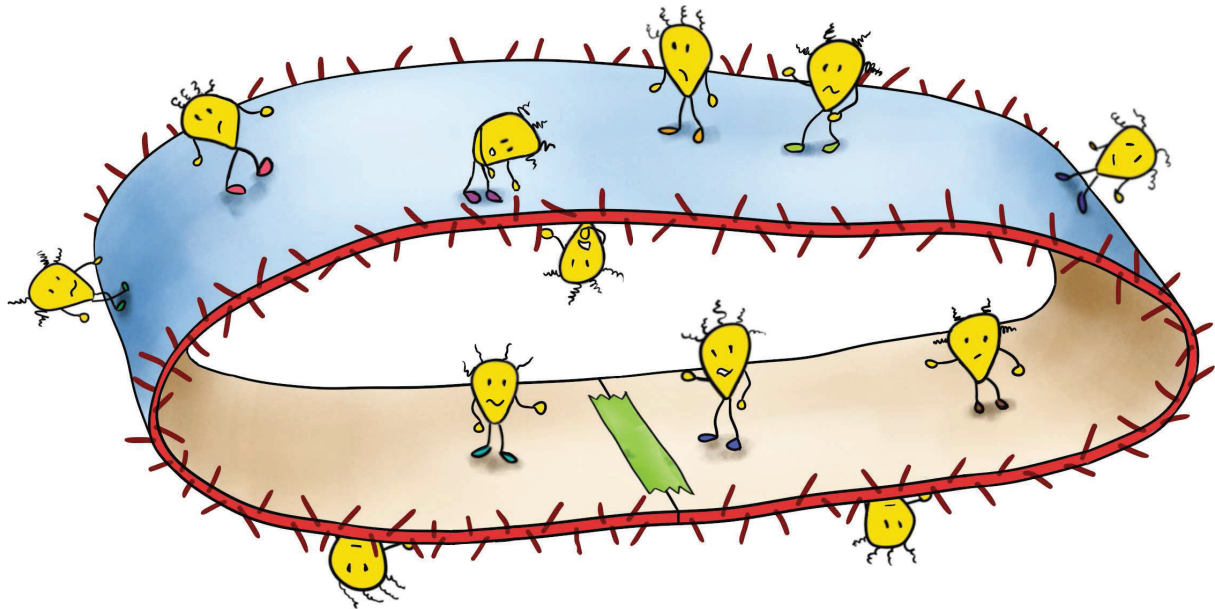
I wrote this fairy tale as an introduction to a mathematical miniature for children about the Möbius strip (Möbius band). Illustrations by Ewa Nadtoczy.

Once upon a time, beyond seven mountains, beyond seven forests and beyond seven galaxies there was a certain small planet. It was called Bendy and had an unusual shape. It was unusual because it was not like a ball but it looked like a partially wrapped strip. On the Bendy lived some small nice creatures. They were joyful, happy and led a well and prosperous life. The planet had a drawback, an inconvenience for its inhabitants. The planet had two sides, blue and brown. Small creatures lived on both sides of the planet. From one side to the other you could get through only in places that were marked green. However, in places where the edge was red and bristled with spikes, the passage was impossible. But residents often crossed the green edge to meet talk and have fun together.



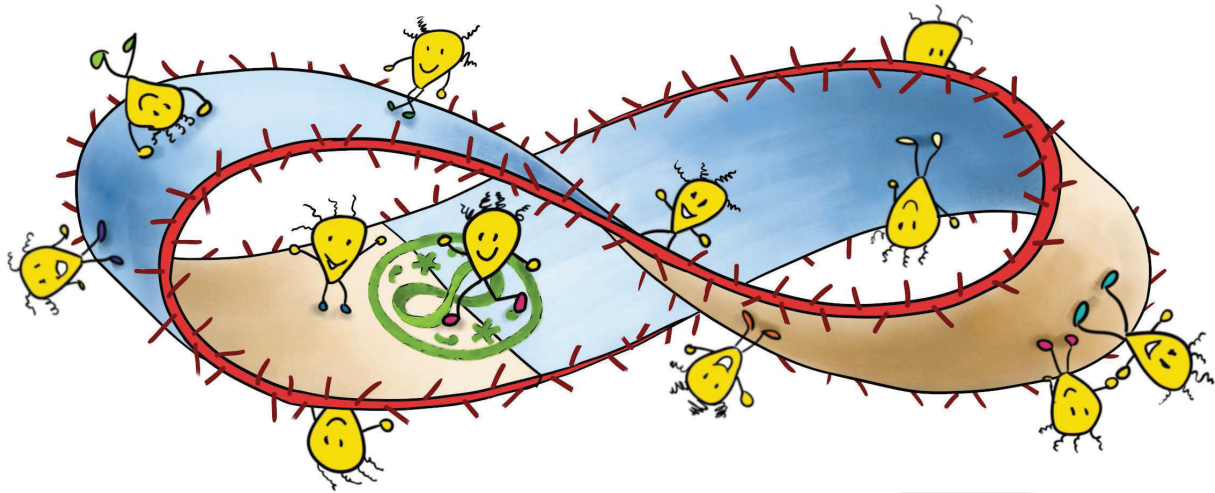
Until one time disaster struck. A terrible monster Suibom came to the area of Bendy. He did not like the fact that the creatures living on this planet ruled themselves in their own way and could move all over its surface, and thus stayed on both one side and the other. The evil Suibom came up with an idea of how to harm the happy residents. He brought powerful equipment and changed the shape of the planet. He connected the two

green edges. Now the inhabitants were separated. There were no green edges anymore, so they could not move from one side to the other. Suibom threatened to destroy the whole Bendy if the residents cut the joining point. He said that henceforth the edges were to be joined forever. Little creatures were not able to meet each other, there was great depression and tears and sadness. Who could save us, they thought. Maybe some knight with a sword, maybe a superman or a batman, or maybe AI?



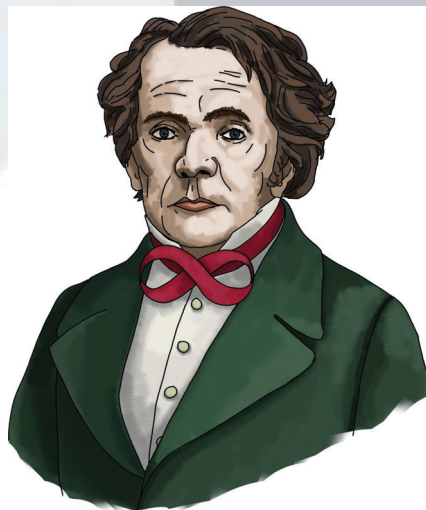
But someone else arrived to the rescue. It was Mobius. He asked if Suibom agreed that Mobius should cut the site of the connection, and then reconnect the green edges of the Bendy. Suibom at first did not want to allow it. He feared that all the inhabitants would move to one side of the planet. But Mobius promised the monster the following thing.

— Mighty Suibom, do not fear that the little creatures will move to one side. I promise you that everyone, from the moment of cutting to the reconnection of the green edges of Bendy, will stay in its place. Besides, it will take a very, very short time, a moment of time and everything will be connected. And at the point of connection I will place a seal that will remain intact forever. After this assurance and receiving the guarantee of the eternal seal, Suibom agreed. Then Mobius did what he said, but. . . after cutting, he performed a trick. He quickly turned one of the edges upside down and only then joined two green edges. He placed a perpetual seal at the connection point, merging the edges forever. The beast watched with satisfaction as the creatures did not move from their places, and the connection was sealed forever. Mobius quickly moved away. The monster wondered what that cut was about. He looked at the Bendy again. He looked and understood – all the inhabitants the planet will be able to walk across its entire surface and meet each other, and he — the Suibom approved the perpetual sealing.



The monstrous beast shook and roared horribly and became so angry that it exploded and dispersed into tiny parts. The letters of the name Suibom scattered and reordered, arranged themselves in. . . Möbius! Where do the dots above the letter o come from? Because the eyes of the dreadful monster became very very small and jumped over the letter o.

The next edition of the bulletin will discuss more about the Möbius strip and related Kangaroo questions. Stay tuned!



August Ferdinand Möbius decorated with a bow tie of his name.

Scripta Manent

The purpose of this column is to discuss, periodically, proverbial phrases from philosophy, literature or history that are relevant to Mathematics. In each case, we explore the origin, meaning, and use of maxims which mathematicians and intellectuals often like to refer to.



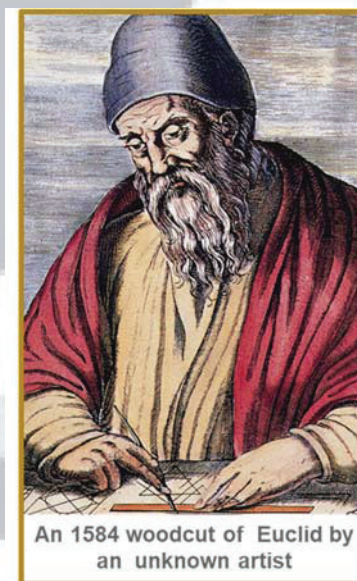
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Οπερ ἐδει δεῖξαι (ο.ε.δ) or Quod erat demonstrandum (Q.E.D.)

The Latin phrase “quod erat demonstrandum”, often abbreviated as Q.E.D., is an exact translation of the corresponding Greek phrase “oper edei deixae”, abbreviated as ο.ε.δ., which comes from Greek Mathematics in antiquity. More on this below. Its meaning is “(being) what it was required to prove”.

Etymologically the word “deixae” comes from the verb “deiknimi” which means “to point out”, “to show”, “to prove”, “to demonstrate”. The phrase itself has been used ever since in mathematical and philosophical texts as the closing sentence to signify that a proof or an argument has been concluded.

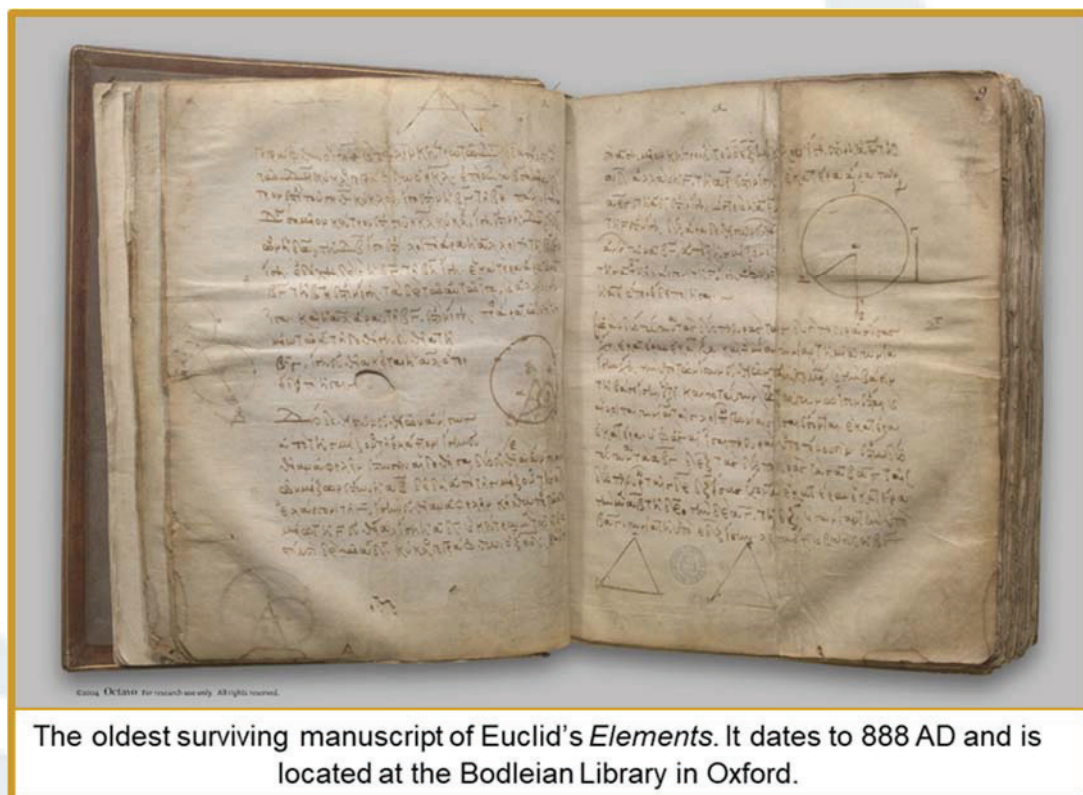
The earliest use of the phrase “oper edei deixae” that we are discussing, is in the celebrated Elements of Euclid which was written around 300 BC, at the Golden Age of Greek Mathematics. Recall that the Elements of Euclid, although not the oldest surviving Mathematical work (there were several others in older civilizations) is certainly the oldest surviving text in the full sense of the word Mathematics, as we understand it today. Namely, with Definitions, Axioms, Lemmas, Theorems, Proofs, Corollaries and, above all, a large body of highly interconnected and important results developed by a strictly logical line of arguments. Each and every one of the 468 Theorems in the Elements is accompanied by a proof and each closes with the phrase “oper edei deixae”. The first occurrence is at the beginning of Book 1, as early as Proposition 4. The first three are prerequisites involving ruler and compass constructions. The next one, which is the very first Theorem in the Elements, is the familiar criterion for equality of triangles. I include it here, in his own words, as I translated them from the earliest manuscript of the Elements, today a treasure of the Bodleian Library in Oxford.



Elements, Book I, Proposition 4.

If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.

A proof of the statement follows adhering to the axioms and logical principles still observed by mathematicians today and, as mentioned, the final sentence is “oper edei deixae”.



During Medieval and Renaissance times almost all the surviving Greek mathematical texts were translated into Latin. These translations were either directly from the Greek original or a translation of an earlier Arabic translation based on the original Greek text. In these early translations, the phrase “oper edei deixae” was rendered not exactly as “quod erat demonstrandum”, which we are discussing here, but in a more or less equivalent form. For example, the 12th-century medieval English scholar Adelard of Bath (ca.1075–ca 1160), who is credited with introducing Euclidean Geometry to the Latin-speaking Western European world, based his translation on Arabic sources. His translation typically ended the proofs with the phrase “ut demonstratum est,” (as has been demonstrated). Elsewhere we see “quod oportet fieri” (which was necessary to be done).

The version “quod erat demonstrandum” of the phrase, more common today, appears in Renaissance times when the invention of printing was paramount for the dissemination of knowledge. The first occurrence is in 1501 in the work “De expetendis, et fugiendis

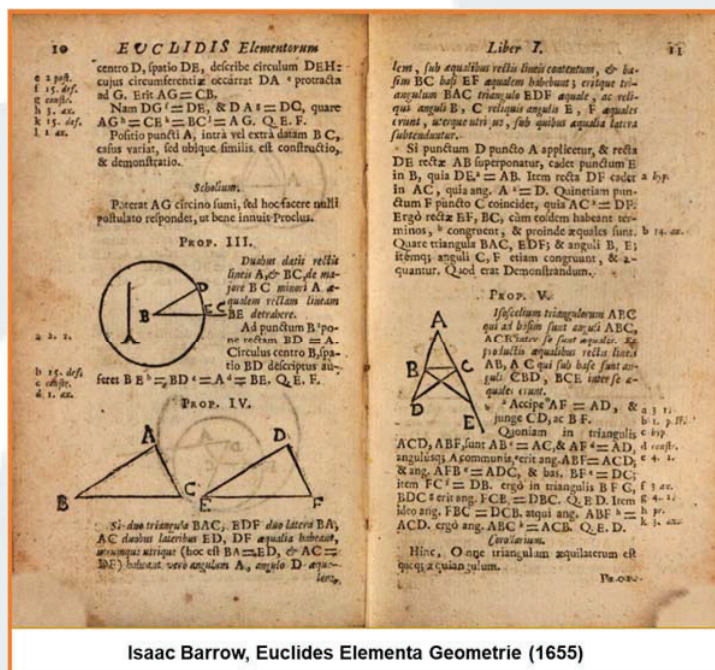
rebus” (On things to be sought and things to be avoided) of the important Italian scholar, mathematician, philologist and translator Giorgio Valla (1447-1500). In this book, he translated, directly from Greek in which he was well-versed, parts of Euclid’s Elements, where he used the phrase we are discussing.

Soon after Valla, and no doubt the book that established the phrase “quod erat demonstrandum” in the vocabulary of mathematicians, was the translation of Euclid’s Elements by the Italian humanist Bartholemeo Zamberti (1473-1543), published in Venice in 1505. A seminal work by all counts.

The abbreviation Q.E.D. was used for the first time in 1598 in the 48-page booklet "Problema, quod iubet ex Quatuor rectis lineis datis quadrilaterum fieri, quod sit in Circulo” (Problem which requires that a quadrilateral be formed from four given straight lines, which is in a circle) written by the Bohemian mathematician and astronomer Johannes Praetorius (or Johann Richter) (1537–1616).

An extensive use of the phrase “quod erat demonstrandum” is by the great English scholar and mathematician Isaac Barrow (1630 –1677), who among other contributions is credited with his work on the early development of infinitesimal calculus. Among his extensive works, he also published in 1655 a widely circulating translation into Latin of Euclid's Elements. The picture below is open on the page that includes Book I,

Proposition 4 (see above). You can see at the end of the proof the phrase “quod erat demonstrandum”. There is yet another phrase that is frequent in Euclid's Elements which is related to “quote erat demonstrandum”. Note that the Elements do not consist only of Theorems and Lemmas which require proofs, but part of them deal with ruler and compass geometric constructions. For example, they contain constructions of bisectors and perpendiculars but also more elaborate ones like the constructions of regular polygons. At the end of every such construction, the Elements conclude with the phrase “oper edei poiesai”, abbreviated o.e.π., which in Latin is rendered as “quod erat faciendum”, abbreviated Q.E.F. They mean “(being) what it was required to do”. The first three propositions in the Elements are such constructions. For example, the opening of the text is



Isaac Barrow, Euclides Elementa Geometrie (1655)

Elements conclude with the phrase “oper edei poiesai”, abbreviated o.e.π., which in Latin is rendered as “quod erat faciendum”, abbreviated Q.E.F. They mean “(being) what it was required to do”. The first three propositions in the Elements are such constructions. For example, the opening of the text is

Elements, Book I, Proposition 1.

On a given finite straight line (segment) to construct an equilateral triangle.

Let me close by mentioning that several Philosophers used extensively the phrases discussed here. Perhaps the most famous book is the Ethica, ordine geometrico demonstrata (Ethics, Demonstrated in Geometrical Order) by Baruch Spinoza (1632–1677). In fact, the whole book, influenced by Descartes, has a mathematical structure, starting with a small number of definitions and axioms from which he derives a large number of propositions and corollaries.

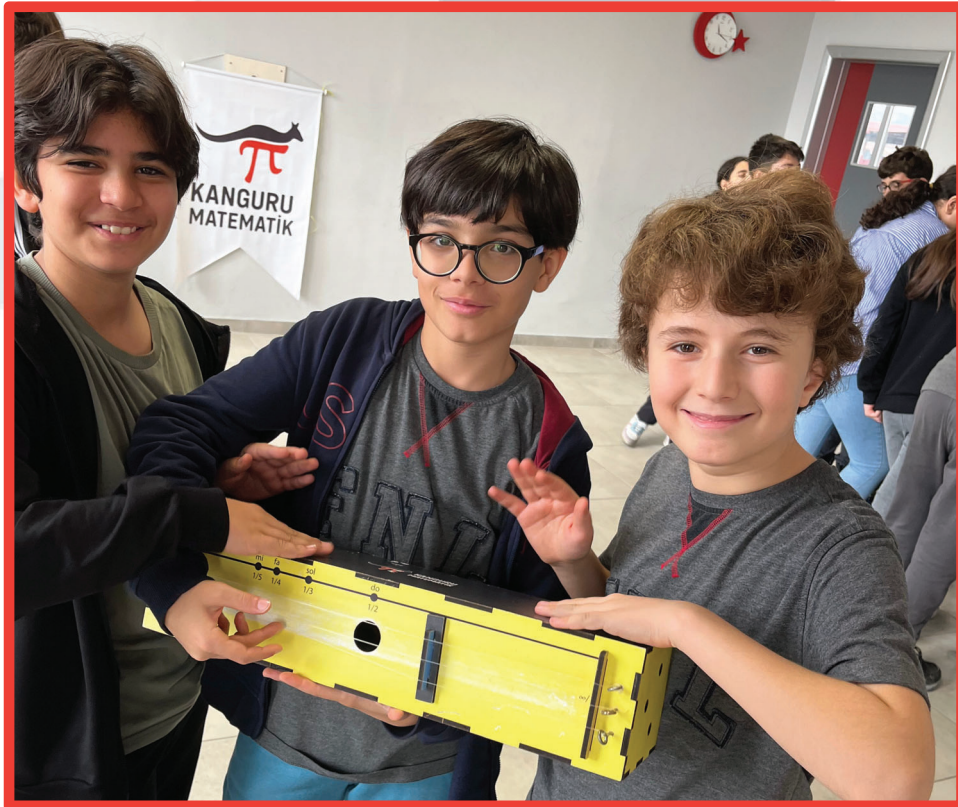
Math Museums: exploring the mathematics...

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Mathematics is often perceived as a difficult and abstract subject, but math museums around the world are changing this perception by making math interactive, fun, and engaging. These museums provide hands-on experiences that allow children to explore mathematical concepts in a creative and stimulating environment, helping them develop a deeper understanding and appreciation for the subject.

One of the biggest advantages of math museums is their ability to make abstract concepts tangible. Through exhibits featuring puzzles, games, and interactive installations, children can visualize and manipulate mathematical ideas, making learning more intuitive. For example, exhibits on symmetry, fractals, probability, and geometry allow students to experiment with patterns, shapes, and numbers in ways that traditional classroom settings may not always provide.



Additionally, math museums help cultivate a sense of curiosity and problem-solving skills. By engaging in hands-on activities, children are encouraged to think critically and explore solutions through trial and error. This experiential learning approach fosters a positive attitude towards math, reducing anxiety and increasing confidence in their abilities.

Math museums also create a social and collaborative learning experience. Unlike traditional lessons that are often solitary, these museums promote group participation, discussions, and teamwork. Whether working together to solve a logic puzzle or competing in a mathematical game, children develop communication and reasoning skills that are essential for their academic and personal growth.

Moreover, these museums inspire future generations by showcasing real-world applications of math. Exhibits highlighting the role of mathematics in engineering, technology, art, and nature help children understand its relevance beyond textbooks. This connection between math and everyday life sparks interest and motivation, potentially leading to careers in STEM (science, technology, engineering, and mathematics) fields.

In conclusion, like our math contest, math museums play a crucial role in transforming the way children perceive and engage with mathematics. By providing interactive, hands-on experiences, they make math more accessible, enjoyable, and inspiring.

Excitingly, a new math museum is set to open soon in Istanbul, offering even more opportunities for children and families to explore the wonders of math through innovative exhibits and engaging activities.

As one of the founders of the new Kangaroo Mathematics Museum, we are proud to be a part of this development. Hoping it will set an example for the establishment of new mathematics museums...



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